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Fuzzy Models of Intelligent Control Systems: Theoretical and Applied Aspects (A Survey)*

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A unified approach is considered to the construction of problem-oriented models of intelligent fuzzy automatic control systems (ACS) used in a variety of industrial fields. Primary attention is devoted to a description of operating fuzzy ACSs and controllers which have many advantages over traditional control systems. The problems of choosing and constructing optimum fuzzy control algorithms are discussed in detail. The problems of realizing control algorithms and controllers based on fuzzy processors and expert systems are analyzed. Sample intelligent automated workstations (AWS) designed by fuzzy processor and controller computer-aided design systems are given.

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INTRODUCTION

The extensive use of robotic and human-computer systems (from industrial flexible manufacturing systems through medical and biological systems) has made it necessary for designers to address the problem of improving the flexibility of ACSs as well as the reliability (fault resistance and survivability) of similar systems [1–3]. Improving such performance factors and quality of a solution by modernizing solely the hardware of the ACS structure has at a certain stage yielded a significant improvement in the automation level of a variety of industrial processes. An analysis of the applications of robotic systems in industry, and particularly in the nonindustrial sphere (such as biomedical systems, program-controlled auxiliary systems for replacing lost functions, etc.) and in critical systems (including control systems for nuclear power plants and atomic reactor diagnostic systems; control systems for pathophysiological processes; on-board spacecraft automatic control systems; automated technological process control systems for petroleum refining industries and other explosion-hazardous industries, etc.) has revealed the existence of limits on the maximum possible attainability of such characteristics in the hardware implementation only. We know that improving automatic control system flexibility by expanding the hardware realization will serve to reduce the fault-resistance and viability of the overall system, etc. The formalization of human operating behavior (accounting for errors) in the corresponding human-computer systems and integrated automated control systems takes on an added significance. Increases in the "intelligence" of the ACSs can be formulated in a manner analogous to [4] as the principle of "decreasing ACS precision by increasing intelligence."

One possible direction for solving this problem involves reducing the level of complexity of the hardware-software package of the automatic control systems by increasing their "intelligence" and creating integrated "reasoning" industries with progressive technology. In robotics complexes and flexible manufacturing systems (FMS) this is achieved by using fuzzy controllers and artificial-intelligence ACSs [5–11], whereas in biotechnology and biomedical systems the functional capabilities of the homeostasis of the integrated structures are also taken into account [12–13]. In such intelligent systems the controllers and decision-makers, as demonstrated in specific examples in [14–17], are realized by models of fuzzy controllers and expert system (ES) functions employing fuzzy instructions and control algorithms, generalized fuzzy logic inference rules, enhanced forms of deep knowledge representation and description in second generation problem-oriented expert systems. The formalization and qualitative interpretation of descriptional adequacy of fuzzy controllers are based on linguistic approximation procedures, the resolution of logic syllogisms, and a structural analysis of fuzzy relations and

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solutions of roughly corresponding multidimensional fuzzy controls using the principle of maximum (minimum) entropy [14]. The degree of complexity of the structural realization and the corresponding software-hardware package is reduced by employing adjustable VLSI microprocessor modules as linguistic processor prototypes of sixth generation fuzzy reasoning computers [9, 13, 18–21]. Between 16 and 64 logic rules (using 2.5-μ fabrication technology) or up to 256 logic rules (using 1.25-μ technology) can be realized on a 6 × 6 mm chip containing 8300 transistor elements fabricated by CMOS-technology. The reasoning speed in this case is $8 \cdot 10^4$ logic rules per second (the reasoning speed today has attained 10^6 logic rules per second); up to 117 rules are handled in real time by parallel processing, and both analog and digital integrated circuits are used for the hardware realization of the processor [9, 18–21]. Designer automated workstations [22–24] for designing logic controllers consisting of fuzzy VLSI processors [25–39] have been developed. Subsequent development of VLSI fuzzy processors involved analysis of computation schemes to implement multidimensional membership and parallel fuzzy reasoning functions for multidimensional real-time multiple input-output ACSs [9, 17, 26, 29, 31, 33, 40–46]. Such research in turn made it necessary to develop a new component base for fuzzy processors [47–51] that implement standard logic fuzzy operations and their combinations. Hence controllers based on fuzzy processors are superior to traditional P-, PI-, and PID-controllers in terms of transfer process quality and controllability [6–8, 10, 14–18, 34–38, 52–54]. Theoretical research and simulation results indicate a broad range of applicability of fuzzy models of controllers and a dependence between *a priori* information and the subject domain [14–17, 55, 56].

In the general case it is possible to analyze modes of dynamical control objects and ACSs by employing structural analysis of fuzzy models [15–17, 57–68] containing fuzzy controllers on feedback connective channels. Such connectives (as an analog of [69]) are formed in weakly structured systems by linguistic approximations of control loops and fuzzy controllers using the principle of two-channel invariance developed by the school of Academician B.N. Petrov. In this case, the first generalized deviation control channel with compensation utilizes a fuzzy controller based on the principles of classical fuzzy logic; the second generalized control and state monitoring channel of the object is designed to account for changes in the information parameters of the state of the object under extremal cases, and utilizes a fuzzy controller with elements of artificial intelligence in which reasoning is achieved by means of a nondistributive algebraic array (quantum fuzzy logic) [70]. One example of this case is a design of an intelligent ACS for an artificial lung machine [13] employing two fuzzy controllers. The breathing loop in this case is controlled by means of the first fuzzy controller which employs classical fuzzy logic; the second nonfuzzy quantum-logic controller monitors the interaction of the respiratory system and the discrete states of the cardiopulmonary system (which the respiratory system is combatting) and corrects the corresponding tables of linguistic decision-making rules. In this case, the knowledge base of the quantum-logic controller accounts for the homeostatic capabilities of the entire body.

This paper presents a survey of the practical application of such aspects of fuzzy model theory to problems of constructing optimum industrial control systems for complex dynamical systems. The problems of analyzing fuzzy algebraic and differential equations, fuzzy measures (such as fuzzy entropy) and their relation to fuzzy operators lie beyond the scope of this paper, as do secondary issues concerning the adaptation and estimation of the sensitivity of the behavior of ACS structures as a function of the type of fuzzy implication operator, descriptions of hardware structures of fuzzy processors and their application in nonfuzzy expert systems; the description of fuzzy models for control of relativistic and quantum dynamical systems [71, 72] and many other aspects. These problems are partially discussed in [19, 14–17, 70–75].

1. Fuzzy Models of Optimum Control Over Dynamical Systems Under Conditions of Uncertainty

The traditional approach to solving problems of optimum control system theory based on formal logic methods currently employed in mathematics is used to create exact (in the broad sense) models of rigorous reasoning and inference. In this case primary attention has been devoted to the problems of correctness, completeness, consistency, closure, stability, controllability, and many other qualitative aspects of a description of models of objects and controlled algorithms. The problem of assessing the truth of statements of logic propositions such as "the mathematical model is an adequate representation of the real control object" essentially remained open and could not be solved solely within the framework of the analysis method used. We have an analogy here to Gödel's incompleteness theorem in arithmetic.

The construction of models of dynamical systems as control objects is one of the fundamental tasks of automatic control theory. This largely involves solving the following problems [14, 76]: descriptions of processes that occur in objects and automatic control systems (ACS); choosing the corresponding methods of formalizing and establishing a correlation (adequacy) between the models obtained in this manner and the initial object as well as the analysis methods (depending on the level of physical and mathematical rigor). Note that the process of constructing models of physical processes is a complex, evolutionary process, and involves an unavoidable approximation of the actual object and will lead to a loss of information in the description of the object. In this case, the hypotheses and axioms employed to approximate and describe a real object by means of a corresponding model cannot take account of all essential aspects of the physical process, which leads to a certain increase in risk and uncertainty in describing the control object.

Estimates of the increase in such risk can be obtained based on an information approach [77]. An assessment of the degree to which a mathematical model provides an adequate representation of an actual control object is arbitrary (relative) in nature and is essentially dependent on the hypotheses used to describe the test dynamical system [14, 76, 78, 79]. Three versions are examined from the viewpoint of fuzzy model theory: 1) fuzziness of a description as an approximation of a weakly structured model of an actual control object due to its complexity and uncertainty of information on its properties; 2) the actual object has an objective internal fuzziness in the description of its operation.

In the first case the estimate of the degree to which the model is an adequate representation of the actual object is established by a fuzzy measure of the relation between comparable systems [80] and simulation methods [14, 16, 17, 81–85]. This approach is particularly valuable in constructing knowledge-based industrial intelligent automatic control systems that employ corresponding artificial intelligence devices in the control loops [9, 16, 17, 73–75, 86–88].

In the second case, studies of the completeness of the corresponding estimate of the adequacy of the description of the control object have demonstrated [70, 89–91] that the truth of its value can only be found in the open interval (0, 1). This result means that there is a class of dynamical systems for which the truth of statements regarding the degree to which systems and the system components are adequately represented by its models fundamentally cannot adopt Boolean values of {0, 1}. It turned out that such a class of dynamical systems is described within the framework of quantum logic [92–95] in which the logic connectives have certain differentiating features from their corresponding classical connectives [94, Chapter 20]. In this case, the class of dynamical systems described by quantum logic contains not only quantum-relativistic control objects, but traditional control objects as well. Examples of such systems can be found in [13, 70, 96]. The last conclusion is due to a continuous quantum-classical logic limit process (unlike the description of the corresponding transitions, which is a discrete transition at the physical level), i.e., there exists an analytic function ρ for the transition from quantum logic to classical logic. In this case the language L_1 in which the quantum logic language L is embedded contains (in addition to the traditional logic connectives) an additional connective: the modal operator P ("possibility") [97]. Therefore, possibility theory is an inclusion in quantum fuzzy logic [70]. The departure point in such research has been the study by Birkhoff and Von Neumann [98] on the adequacy of physical and logic levels for describing models of quantum systems. These results indicate that the logic description level in some sense is broader than the physical level [70] since it is applied to a description of a broader class of dynamical systems. Such results are significant in developing space-time pseudophysical logic [99]. One example is the problem of control over the group motion of an ensemble of independent vertical-displacement mobile robots employing industrial manipulators under extremal conditions (such as fire-fighting robots in explosion-hazardous media, as well as for cleaning and deactivation of surfaces, etc. etc.) [100–103].

This paper is largely limited to the first version of analyzing fuzzy models of industrial automatic control systems.

Let us consider a possible qualitative approach to analyzing control processes over complex systems based on the principles of fuzzy logic [9, 4, 73–75], while the set of control algorithms will be used in a given class of fuzzy systems [16, 17, 75].

The state of the complex system and the control actions are considered to be linguistic variables, while the specific control values are chosen based on a compositional inference rule [9, 14, 74].

1.1. **Primary definitions of fuzzy set theory and fuzzy logic.** Here we list the primary definitions required for presenting the fundamental results of this section.

Definition 1. Let X be a set of arbitrary nature. The set of ordered pairs $(x, \mu_A(x))$, where $x \in X$, $\mu_A(x) \in [0, 1]$ for all x is called the fuzzy subset A of X. The function $\mu_A(x)$ is called the membership function and can be treated as knowledge of the degree of membership of the element x in fuzzy-defined set A.

Standard set-theoretic unification, intersection, and negation operations can be performed on fuzzy subsets of x:

$$\begin{cases} \mu_{A \cup B}(x) = \mu_A(x) \bigvee \mu_B(x) = \max\{\mu_A(x), \mu_B(x)\}; \\ \mu_{A \cup B}(x) = \mu_A(x) \bigwedge \mu_B(x) = \min\{\mu_A(x), \mu_B(x)\}; \\ \mu_{\neg A}(x) = 1 - \mu_A(x), \end{cases}$$

as well as, for example, an involution operation

$$\mu_{A^{\alpha}}(x)=\mu_A{}^{\alpha}(x) \text{ for } \alpha>0.$$

A more comprehensive mathematical description of the logic connectives of fuzzy set theory from the position of t-norms and t-conorms can be found in [104].

A fuzzy subset is called a normal subset if $\sup\limits_x \mu_A(x) = 1$. The set of fuzzy subsets X is called a fuzzy subalgebra of X and is denoted by $F(X)$.

Definition 2. The ordinary (clear) set defined by the expression $Y_a = \{y \in X, \mu_X(y) \geq a\}$ is called the set of level X_a, $a \in [0, 1]$ of the fuzzy subset Y in X and is denoted by Y_a.

Definition 3. Let X be a space with the measure $\nu(\cdot)$ such that $0 < \nu(x) < \infty$ and $A \in F(X)$. The functional

$$\alpha(A)=\frac{1}{\nu(\operatorname{supp} A)}\int_x \Delta(\mu_A(x))\,d\nu(x), \tag{1.1}$$

where the entropy $\Delta(z) = -z \ln z - (1-z)\ln(1-z)$; $\operatorname{supp} A = \{x, \mu_A(x) > 0\}$ is called the degree of smearing of fuzzy set A. We can easily test Eq. (1.1) for satisfaction of the following properties: 1) $\alpha(A) = 0$ when and only when A is an ordinary (nonclear) set; 2) $\alpha(A) = \alpha(\neg A)$; 3) $\alpha(A) + \alpha(B) = \alpha(A \vee B) + \alpha(A \wedge B)$; 4) $\alpha(A)$ reaches a maximum for $|A| \equiv \frac{1}{2}|X|$; 5) If A and A^* are such that

$$\begin{cases} \mu_{A^*}(x)\geqslant\mu_A(x), \text{ for all } x, \text{ where } \mu_A(x)\geqslant\dfrac{1}{2}, \\ \mu_{A^*}(x)\leqslant\mu_A(x), \text{ for all } x, \text{ where } \mu_A(x)\leqslant\dfrac{1}{2}, \end{cases}$$

then $\alpha(A^*) \leq \alpha(A)$, which is in agreement with intuitive notions of the degree of uncertainty of situations described by fuzzy subset A. Note that the following expansion is valid:

$$F(X)=\bigcup_{\lambda\geqslant 0} F_\lambda(X), \tag{1.2}$$

where $F_\lambda(X) = \{A \subseteq F(X); \alpha(A) \leq \lambda\}$, $F_0(X) \equiv X$. It should be emphasized that in expansion (1.2) the sets of level λ will depend on the established radius (measure) of the uncertainty $\alpha(A)$ in accordance with Eq. (1.1) defined by the degree of information available to the researcher: the corresponding entropy. This approach establishes the limits on the applicability of these models as measures of the adequacy of representation of the actual control objects.

Definition 4. The linguistic variable is an ordered set $(S, X, T(s), G, M)$, where S is the variable name; X is the base set of values of the variable; $T(s)$ is the term-set of the linguistic variable s which is a family $\{\bar{X}_i\}_{i=1}^n$ of normal fuzzy subsets X such that $\bigcup\limits_{i=1}^{n} \operatorname{supp} X_i = X$; G is the contextually-free grammar generating the set of all values of S on $T(s)$; M are the rules for calculating the membership function of the composite value of S from values of $T(s)$.

Definition 5. The fuzzy subset with the given membership function $\mu_f(x, y)$ is called a fuzzy mapping $\bar{f}: X \to Y$ of set X on to set Y. Note that fuzzy mapping $\bar{f}$ is in fact not only defined on X but also on $F(X)$. If $\{A, \mu_A(x)\} \in F(X)$, then $\mu_{f(A)}(y) = \bigvee\limits_X [\mu_A(x) \wedge \mu_f(x, y)]$. Naturally $\operatorname{Im} f \subseteq F(Y)$.

The simplest example of a fuzzy mapping is the following subset $X \times Y$:

$$\mu_f(x, y)=[\mu_A(x)\wedge\mu_B(y)]\vee[\mu_{\neg A}(x)\wedge\mu_C(y)],$$

where $A \in F(X)$; $B, C \in F(Y)$.

In such a case we can easily test the validity of the equalities $\bar{f}(A) = B$ and $\bar{f}(\neg A) = C$ which yields a basis for assigning to $\bar{f}$ the linguistic label: **if** A, **then** B, **else** C.

A somewhat more complex example is a fuzzy mapping with the properties $\mu_f(x, y) = \bigvee_{i=\overline{1,n}} (\mu_{A_i}(x) \wedge \mu_{B_i}(y))$, where

$$\sum_{i=1}^{n} \mu_{A_i}(x) = 1.$$

In this case

$$\bar{f}(A_1) = B_1, \ldots, \bar{f}(A_n) = B_n,$$

which provides a basis for assigning to $\bar{f}$ the linguistic label

$$\begin{cases} \text{if } A_1 \text{ then } B_1 \text{ else} \ldots \\ \cdots\cdots\cdots\cdots \\ \text{if } A_n \text{ then } B_n. \end{cases} \tag{1.3}$$

Consistent with the definition provided in [14, 105] we will call the number $C(\bar{f}) = \ln(n)$ the computational (algorithmic) complexity of mapping $\bar{f}$.

These definitions make it possible to formulate the following primary results.

1.2. **Fuzzy differential inclusions and optimum control processes.** A variety of methods [106–107] have been used to analyze the problem of control of dynamical systems under conditions of uncertainty. Here we will consider only one possible approach based on differential inclusion theory [108, 109] generalized to fuzzy systems [110–113], as yielding the most adequate description of the behavior of dynamical systems under conditions of uncertainty. Let us assume that the behavior of the dynamical system is described by the differential equations

$$\dot{x} = f(t, x, k), \tag{1.4}$$

where k is the parameter vector on the right side of the equation. In the general case, vector k is unknown and may vary arbitrarily. In practice we often know a set K to which possible values of $k \in K$ belong. In this case Eq. (1.4) is best replaced by the differential inclusion

$$\dot{x} \in F(t, x, K). \tag{1.5}$$

If different points in the set U are not equally valid as possible realizations of K, the set K can be called a fuzzy set [110–113]. According to Zadeh's generalization principle the function $f(t, x)$ is extended to the family of fuzzy sets $F(\cdot)$, i.e., we obtain a fuzzy set on the right side of Eq. (1.5). It turned out to be possible to introduce the concept of solving a fuzzy differential inclusion of the (1.5) type through the concept of Y_a: the set of level a. Different definitions of this concept and its modifications can be found in [111, 114, 115].

Here we briefly consider the relation of fuzzy differential inclusion theory to the method of "viability" theory [113, 116] and its application to determining optimum equations [117] described by "viscous" solutions of the Hamilton-Jacobi-Bellman equations [118–122].

In fuzzy differential inclusion theory the "fuzzy dynamic" of the test control system with feedback of the type $\dot{x} = f(t, x, u(x))$ reduces to $\dot{x} \in F(x), x \in X, u \in U(x)$ and is replaced by a fuzzy graph [9] described by the membership function $\mu_F(x, \dot{x})$: $X \times X \to R_+ \cup \{+\infty\}$. In this case there exists a cost function of the control goal V: $X \to R_+ \cup \{+\infty\}$ in which the domain of existence $\mathrm{Dom}\{V\} := \{x \in X \mid V(x) < \infty\}$ and $\mathrm{Dom}(V) \subset \mathrm{Dom}(\mu_F(x, \dot{x}))$.

We introduce the expression

$$D_\dagger(V)(x)(u) := \lim_{h \to 0_+} \inf_{u' \to u} [V(x + hu') - V(x)]/h. \tag{1.6}$$

Expression (1.6) is called the tangential epiderivative of the function V at point x in direction u. We denote by $T_k(k)$ the Bouligand tangential cosine defined as

$$T_K(x) := \{\dot{x}=v\in X \mid \liminf_{n\to 0_+} d_K(x+hv)/h=0\}, \tag{1.7}$$

where $d_K(y) := \inf_{z\in K} \| y - z \|$ is the distance from y to K. The properties of the function $V(x)$ are described by the epigraph

$$\mathrm{Ep}(V) := \{(x,\dot{\lambda})\in X\times R \mid V(x)\leqslant\lambda\}, \quad \lambda = \sup_{x\in K} \inf_{v\in T_K(x)} \mu(x,v)<\infty.$$

Let us consider the auxiliary differential equation

$$\dot{w}(t)=-\varphi(w(t)),\ w(0)=V(x(0)), \tag{1.8}$$

whose solution $w(t)$ approximates the behavior of the function V as $\forall t > 0$, $V(x(t)) \le w(t)$.

We define the tangential cosine $T_V^\varphi(x)$ of the type in Eq. (1.7) as

$$T_V^\varphi(x) := \{v\in X \mid D_\uparrow V(x)(v)+\varphi(V(x))\leqslant 0\}. \tag{1.9}$$

Then the non-negative function V with tangential epiderivative (1.6) is a Lyapunov function that is associated with the function φ in Eq. (1.8) if and only if the function satisfies the "viscous" solution of the Hamilton-Jacobi-Bellman equation at the tangential cosine $T_V^\varphi(x)$ [12, 113, 116], i.e.,

$$\forall x \in \mathrm{Dom}(V), \quad \inf_{v\in F(x)} D_\uparrow V(x)(v)+\varphi(V(x))\leqslant 0. \tag{1.10}$$

It follows that a description of a controlled dynamical system with feedback as a fuzzy differential inclusion in which the right side is a fuzzy subset with the membership function $V \in [0,\infty]$ and defined as a cost function reduces to an analysis of the "viscous" solutions of the corresponding Hamilton-Jacobi-Bellman equation of optimum control theory. Possible approaches to constructing solutions of equations of the (1.10) type have been considered in [118–126].

Here we define the bundle of fuzzy trajectories $x(t) \in \mathrm{Dom}(V)$ having the property of "viability" in the sense

$$\forall x\in K,\ F(x)\cap T_K(x)\neq\varnothing. \tag{1.11}$$

Consequently, within the set of solutions of the fuzzy differential inclusion $\dot{x} \in F(x)$ it generates in the sense of solutions (1.10) a bundle (1.11) of trajectories $x(t)$ with the membership function $V(x)$ describing the attraction domain (attractor) under given initial motion conditions of the controlled system. The established relations between the Hamilton-Jacobi (Issacs-Bellman) and Hamilton, Issacs-Bellman, Rosonauer, Pontryagin and the Krotov optimality principle [127] can be used to extend these results to the broader class of controlled dynamical systems.

Let us now consider the problem of a linguistic approximation (LA) and construction of optimum fuzzy control algorithms for corresponding fuzzy motion trajectory controllers for a controlled dynamical system of the type (1.10), (1.11).

We formulate without proof the following theorems [13, 128].

Theorem 1. Let $\tilde{X}_1, \ldots, \tilde{X}_n$ be normal fuzzy subsets of X such that:

1) $\tilde{X}_k \in F_\lambda(X)$ for $\forall k = 1, \ldots, n$; 2) $\bigcup_{k=1}^{n} \operatorname{supp} X_k = X$ and s is the linguistic variable for which $\{\tilde{X}_k\}$ are the term-sets.

Then there exists a value $A^* = LA(A/F_\lambda(X))$ of linguistic variable s for $\forall\varepsilon > 0$ and $\forall A \subseteq F_\lambda(X)$ such that $\rho(A, A^*) \le \varepsilon$.

Theorem 2. Let $\tilde{f}: F(X) \to F(Y)$ be an arbitrary fuzzy mapping of the fuzzy subalgebra $F(X)$ on to $F(Y)$. Then for $\forall\varepsilon > 0$ there exists a fuzzy mapping $\tilde{f}_\varepsilon: F(X) \to F(Y)$ of the type (1.3) such that $\rho(\tilde{f}, \tilde{f}_\varepsilon) \le \varepsilon$.

Theorem 3. Let $f_0: X \to Y$ be an arbitrary mapping of the set X on to set Y. Then, for any fuzzy mapping $k: Y \to F(Y)$ such that $\alpha(k) = \lambda$ there will exist a fuzzy mapping $\tilde{f}_\lambda: X \to F(Y)$ of the type (1.3) such that:

1) $\rho(f(A), \tilde{f}_\lambda(A)) \leq \lambda$ for $\forall A \subset X$;

2) $C(\tilde{f}_\lambda) \approx 0\,(E_{\frac{\lambda}{2}}(X))$, where

$$E_{\frac{\lambda}{2}}(X) - \frac{\lambda}{2} \quad \text{is the potency of set } X.$$

Let us consider the following classical problem of automatic control theory. Let L be a certain dynamical system whose transfer functions will vary depending on operating mode; it is necessary to complete L by means of feedback f such that the transfer functions $L \cup f$ will be optimum in the sense of a given criterion (such as speed). As we well know from optimum control theory, for a broad class of dynamical systems the principle of maximum optimum control

$$f_0:\ \delta \to U, \tag{1.12}$$

where δ is the state space and U is the space of control actions.

Mapping (1.12) uniquely induces the mapping

$$\tilde{f}: F(S) \to F(U),$$

defined on $F(S)$ by the following formula: if $A \in F(S)$ and

$$A = \int_S \mu_A(S)/S, \text{ then } \tilde{f}(A) = \int_S \mu_A(S)/f(S). \tag{1.13}$$

According to Theorems 2, 3, for mapping $\tilde{f}$ for $\forall\lambda \geq 0$, there exists a linguistic approximation $\tilde{f}_\lambda = \text{LA}[\tilde{f}/F_\lambda(S)]$ of the type (1.3) that has a somewhat slower speed than the optimum speed (i.e., $\tilde{f}_0$ in accordance with Eq. (1.12)), yet also has a significantly lower computational complexity $C(\tilde{f}_\lambda)$ (dependent solely on S and λ but independent of f). Therefore in the case of especially complex systems it is natural to search for feedback f in the class of fuzzy mappings $S \to U$ of the type (1.3) rather than in a class of ordinary mappings.

It follows from Theorem 3 and this example that the use of fuzzy mappings makes it possible to reduce the computational complexity of a system.

Let us consider a method of determining a linguistic approximation LA that makes it possible to reduce the algorithmic complexity of the system.

In this case for dynamical systems L represented analytically it is necessary to solve the following variational problem: for a given system L, criterion I and algorithmic complexity constraint C it is necessary to find the fuzzy mapping $f^*: S \to U$ such that

1) $f^* \in H_C = \{f:\ S \to U,\ C(f) \leq C_0\}$,

2) $\min\limits_{f \in H_C} I[(L \cup f) \circ x] = I[(L \cup f^*) \circ x], \quad \forall x \in S.$

In the general case it is natural to use iterative methods to find mapping $\tilde{f}_\lambda$, i.e., to consider learning control systems $\tilde{f}_\lambda(k) \xrightarrow{\text{weak}} \tilde{f}_\lambda$, which leads to a generalization and extension of the theory of learning systems.

The system learning algorithm is realized as follows: (i) the O-iteration of $\tilde{f}_\lambda(0)$ is selected: a linguistic algorithm of the type:

if $\tilde{s}_1$ then U_1 else

if $\tilde{s}_2$ then U_2 else

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if $\tilde{s}_n$ then U_n,

where $\{s_i\}_{i=1}^n$, $\{U_i\}_{i=1}^n$ is the family of normal fuzzy subsets belonging to $\{F_\lambda(S) \cup F_\lambda(U)\}$ and completely covering S and U respectively; (ii) the values of the linguistic variable U generated by $\{U_i\}$ contains the optimum linguistic approximations:

$$\text{LA}[f(\tilde{s}_1), \ldots, \text{LA}[f(\tilde{s}_n)].$$

The existence of such an optimum approximation is guaranteed by Theorem 1. The approximations can be found by the steepest descent method from the graph of the linguistic variable U (for example, by the branch and bound method); (iii) if control by means of the algorithm

$$\bar{f}_\lambda = \begin{cases} \text{if } s_1 \text{ then } f(s_1) \text{ else} \\ \text{-----------} \\ \text{if } s_n \text{ then } f(s_n) \end{cases}$$

has a unsatisfactory accuracy, the learning unit will go to a finer partition:

$$s_1^2,\ \rceil s_1^2 \vee \rceil s_2^2,\ s_2^2,\ \rceil s_2^2 \vee \rceil s_3^2, \ldots, s_n^2,$$

which already lies in $F_{\frac{\lambda}{2}}(s)$.

This naturally improves the accuracy of the new algorithm, although its algorithmic complexity $C(f)$ also increases.

Using the method from [77] and the fuzzy entropy and information measures [14, 128] we can show that in this case the most valuable information on the parameter λ makes it possible to reduce the algorithmic complexity.

In this case the fuzziness of a description of finite volume is compensated by the value of the information.

These results demonstrate:

1) systems $\bar{f}_\lambda$ are the simplest systems (from the viewpoint of algorithmic complexity) capable of executing control L for a given dynamical accuracy λ:

2) these learning algorithms make it possible to construct system control processes of a rather high level of complexity by iterative modification of the parameter λ for which traditional methods are either unsuitable or yield a slow convergence of the algorithms;

3) the iterative learning process of the fuzzy systems has a high rate of convergence on $\bar{f}_\lambda$;

4) the strategy $\bar{f}_\lambda$ has the property of invariance (insensitivity) with respect to λ (λ-invariance) and can (as $\lambda \downarrow \infty$) arbitrarily approximate the absolute invariance condition.

This method was tested experimentally for a class of complex dynamical systems simulating the behavior of certain aircraft. The problem of closing a typical sixth-order linear dynamical system by means of feedback was examined. The authors of [14, 128] list the results from computer modelling of the transition processes of this system for $\lambda = 0$ (Feldbaum-optimum) and for specific given values of λ.

One feature of the experiment was the fact that the control object in the simulation algorithm was represented as a module whose description would vary as a function of the object and its operating conditions. The control system itself remained unchanged externally during the simulation process, and automatically adapted to the new object.

Other examples of modelling fuzzy automatic control systems can be found in [16, 17, 73–75, 83–85].

The choice of the fuzzy logic reasoning procedures also plays a significant role in developing methods of designing structures of fuzzy controllers and automatic control systems. This is reflected in the design of hardware systems and in the debugging of software in fuzzy processors. We therefore briefly consider certain features of fuzzy logic reasoning models.

1.3. **Fuzzy logic reasoning and models of a fuzzy implication operator.** The development of the theory of approximate statements and fuzzy reasoning as well as the theory of fuzzy algorithms made it necessary to analyze and synthesize fuzzy logic models associated with the implication operation [16, 17, 70, 129–134]. This can also be explained by the fact that an approach to the representation of knowledge of system elements and control objects as a set of reasoning rules using simulation of dynamical systems behavior has become popular in recent years. The logic simulation of the behavior of a control object automatically produces a knowledge base in the form of a production system "IF (initial situation) and (strategy) THEN (global regularity character)," i.e., as an implication operation. In this case, the simulation algorithm is specific in the sense of fuzzy reasoning and semantic equivalence [135].

Let us consider the modus ponens fuzzy reasoning model widely used in fuzzy controllers:

Premise 1 (Ant 1): if x is A then y is B

Premise 2 (Ant 2): x is A'

Conclusion (cons): y is B'.

Conclusion B' is obtained from Ant 1 and Ant 2 by the application of the max-min-composition rule ($\circ$) to the fuzzy set A' and the fuzzy relation (implication) $A \to B$:

$$B' = A' \circ (A \to B), \text{ e.g. } \mu_{B'}(u) = \bigvee_{n} \{\mu_{A'}(u) \wedge \mu_{A \to B}(u, v)\}.$$

In the particular case when the fuzzy set A is the singleton u_0 ($\mu_{A'}(u_0) = 1, \mu_{A'}(u) = 0$ for $u \neq u_0$), conclusion B' becomes simpler: $\mu_{B'}(v) = \bigvee_u \{\mu_{A'}(u) \wedge \mu_{A \to B}(u, v)\} = \bigvee_{u (u \neq u_0)} \{0 \wedge \mu_{A \to B}(u, v)\} : \bigvee_{u_0} \{1 \wedge \mu_{A \to B}(u_0, v)\} = \mu_{A \to B}(u_0, v)$.

This result demonstrate the role of fuzzy implication and its effect on fuzzy logic reasoning. Cao and Kandel [130] provide the types of fuzzy implication most commonly encountered in the literature. A more detailed description of other types of implication can be found in [16, 17].

The implication operation can also be interpreted as a logic operation in various senses of carrying out logic connective operations on the corresponding membership functions μ_A and μ_B, thereby strengthening or weakening the requirements on the logical conclusion of the implication: "if A, then B." This issue is examined in greater detail in [104]. Here we simply note that the arithmetic Zadeh rule is obtained from the fuzzy Lukasevich implication as a particular case $u = u_0$ (the singleton). Analogously, for $u = u_0$ the Zadeh max-rule follows from the EZ implication (Early Zadeh), while the Bandler implication follows from the Wilmont implication, the modified Bandler implication follows from the Clini-Daynes-Lukasevich implication, etc. Versions of the generalized parametric representation of an implication of the type [136]

$$\mu_{A \to B} = \begin{cases} \mu_A \wedge \mu_B, & \text{for } t=0 \\ \mu_A \cdot \mu_B, & \text{for } t=1 \\ \max(0, \mu_A + \mu_B - 1), & \text{for } t=\infty \\ \log\left(1 + \dfrac{(t^{\mu_A}-1)(t^{\mu_B}-1)}{t-1}\right), & \text{for } 0<t<1 \end{cases}$$

are also possible. Therefore, for $t = 0, 1\ \infty$ we obtain known [130] implications.

Let us now assume that the rule "if . . . then . . ." is related by the logic connective "AND" and that it contains two fuzzy rules "x is A" AND "y is B":

Ant 1: If x is A & y is B, then z is C

Ant 2: x is A' & y is B'

cons: z is C'

where A, A' are fuzzy sets on U, B, B' are fuzzy sets on V, and C, C' are fuzzy sets on W.

In this case, conclusion C' can be obtained from premises Ant 1 and Ant 2 by the application of the max-min-composition rule (*) in the fuzzy sets (A' and B') on the Cartesian product $U \times V$ in the fuzzy implication (A and B) $\to C$ and $U \times V \times W$. Therefore we have

$$C' = (A' \& B') \circ [(A \& B) \to C], \text{ e.g,}$$
$$\mu_{C'}(w) = \bigvee_{u,v} ([\mu_{A'}(u) \wedge \mu_{B'}(v)] \wedge [\mu_A(u) \wedge \mu_B(v)) \to \mu_C(w)].$$

The generalization of these fuzzy logic reasoning rules is an approximate reasoning scheme in which the fuzzy logic premises Ant i ($i = 1, 2, \ldots, n$) are related by the logic connective *else* of the type

Ant 1: If x is A_1 & y is B_1, then z is C_1 else

Ant 2: if x is A_2 & y is B_2, then z is C_2 else

....

Ant n: if x is A_n & y is B_n, then z is C_n

Ant ($n + 1$): x if A' & y if B'

const: z if C'

The following transforms should be noted here [83–85]

$$(A'\&B') \circ [(A\&B) \to C] = [A' \circ (A \to C)] \cap [B' \circ (B \to C)],$$
$$(A'\&B') \circ [(A\&B) \to C] = [A' \circ (A \to C)] \cup [B' \circ (B \to C)].$$

Different signs $\cap$ and $\cup$ are on the right sides of these transforms, which characterizes the differentiating characteristics of the models of fuzzy implication operators discussed in [16, 17, 130]. The first of these equalities holds for the Mamdani-Larsen implications and the limited product, i.e., the logic intersection $\cap$ is used. The second equality holds for the arithmetic

Zadeh rule, the Boolean and Bandler implication, and the standard sequence, i.e., the unification $\cup$ is used.

Therefore the connective *else* takes two forms depending on the fuzzy implication model: $\cap$ and $\cup$.

In the first case when the connective "else" is replaced by the operation $\cap$, the logic conclusion C' (for, for example, the Mamdani implication) takes the following form

$$C' = (A' \& B') \circ [((A_1 \& B_1) \to C_1) \cup \ldots \cup ((A_n \& B_n) \to C_n)] =$$
$$= [(A' \circ A_1 \to C_1) \cap (B' \circ B_1 \to C_1)] \cup \ldots \cup [(A' \circ A_n \to C)_n \cap$$
$$\cap (B' \circ B_n \to C_n)].$$

For the particular case of the singleton $A' = u_0$ and $B' = v_0$, the logic conclusion C' for the Mamdani implication takes the form $C' = C_1' \cup C_2' \cup \ldots \cup C_n'$ and $\mu_{C_i'}(w) = \mu_{A_i}(u_0) \wedge \mu_{B_i}(v_0) \wedge \mu_{C_i}(w)$. Then $\mu_{C'}(w) = [\mu_{A_1}(u_0) \wedge \mu_{B_1}(v_0) \wedge \mu_{C_1}(w)] \vee \ldots \vee [\mu_{A_n}(u_0) \wedge \mu_{B_n}(v_0) \wedge \mu_{C_n}(w)]$. We obtain relations for the logic conclusion C' by substituting the logic connective *else* by the unification $\cup$ analogously. An analysis of more complex situations can be found in [9, 16, 17, 75, 83–85, 137, 138].

2. Fuzzy Models of Controllers and Dynamical Control Systems

In this section a qualitative description of the design principles of fuzzy automatic control models is considered based on the design of fuzzy logic controllers (FLC) widely used in robotic complexes, automated technological process control systems, flexible manufacturing systems for controlling complex dynamical systems, etc. A comparison of these models to traditional objects of control system theory makes it possible to demonstrate a number of advantages and features of the application of fuzzy controllers in industrial automatic control systems.

2.1. **The principles of designing fuzzy intelligent controllers.** Fuzzy logic controllers employ at their base the concepts of fuzzy logic models: fuzzy implication and compositional logic reasoning models. The features of fuzzy logic reasoning are examined in Section 1.3. Here we note that the following linguistic description scheme is traditional for fuzzy controllers utilizing a corresponding production fuzzy logic reasoning model based on a fuzzy production process [9, 18]: fuzzy implication, fuzzy modifiers, fuzzy logic connectives, composition deduction rule, and conversion operators to precise knowledge (defuzzifiers). The construction of a "knowledge base" using knowledge representation and retrieval methods represents the base for designing intelligent controllers. Therefore, industrial fuzzy controllers are based on the principles of artificial intelligence which has recently undergone rapid development [9, 11, 15–17, 86–88, 138, 140].

The structure of an intelligent control system with a fuzzy controller can be represented in accordance with [15, 67, 88] as in Fig. 1. The output variable of control object *1* is industrial process y which is compared to its given value g, with the mismatch error ε input to both the scale element *3* with coefficient K_ε and to the differentiator, whose output is multiplied by $K_{\dot\varepsilon}$ in scale element *5*. Elements *6, 7* are designed to convert the present mismatch value ε and the mismatch derivative (the rate of change in the mismatch) into their linguistic values. The fuzzy values ε^*, $\dot\varepsilon^*$ are input to the principal element of the fuzzy controller: the knowledge base (KB). As a rule, the knowledge base of fuzzy controllers is constructed on the basis of a production knowledge model with an "if . . . then . . ." structure. Each production, which is a set of "situation-action" pairs, can be used to correlate controller action with the evolving situation by assigning values to the control action on the object. Different models of such production rules are examined in [9, 14–17, 83–85].

The linguistic value of the control, after multiplication by the scale coefficient K_u in element *8* and conversion to its exact value u, is presented to the actuator element of the control object.

In this case, the linguistic synthesis of a fuzzy controller can be carried out in accordance with the following two schemes [15, 67]:

$$(\textit{fuzzy model of control object}) \times (\textit{fuzzy controller}) = (\textit{desired closed system}) \qquad (2.1)$$

$$(\textit{fuzzy model of control object}) \times (\textit{fuzzy comparator}) \times (\textit{desired fuzzy closed system}) = (\textit{fuzzy controller}). \qquad (2.2)$$

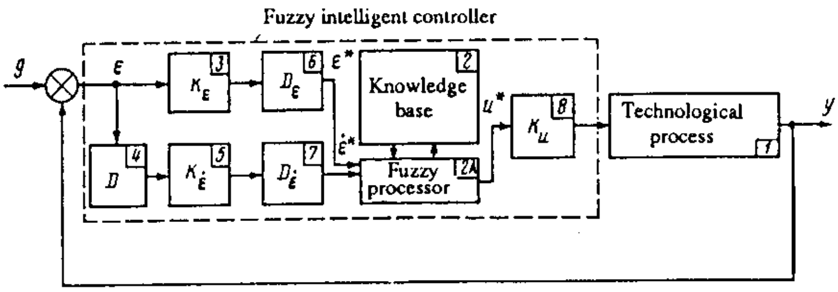

Fig. 1

According to [15] scheme (2.1) is more convenient for linguistic synthesis, while scheme (2.2) is more convenient for linguistic analysis of a closed control system.

Examples of linguistic synthesis can be found in [15, 17]. It is most convenient to use structural geometrical analysis and decomposition methods [17, 57–68, 141–145] for a linguistic analysis of closed automatic control systems.

The primary problem in synthesizing fuzzy controllers is the construction of their knowledge base. The experience and knowledge of the human operator (expert) can be introduced into the knowledge base by the following methods [15, 86–88, 146]: the expert-operator controls the technological process which is "observed" by the controller, while the controller records all expert actions and thereby fills up its knowledge base; the expert-operator formulates his action in each observed situation as an "if . . . then . . ." production whose set comprises the contents of the controller knowledge base; a self-organizing fuzzy controller is assigned to generate a given transfer characteristic of an industrial control system under design; certain information on the technological process—the control object—is simultaneously conveyed. The control independently (by trial and error) accumulates knowledge without the expert.

The following FLC structures were constructed on the basis of this classification.

It should be noted that cause and effect relations for the knowledge base can be generated by means of methods other than production rules. A typical example is knowledge base formation for medical expert systems [147].

In the general case knowledge representation in control systems in artificial intelligence theory is achieved by means of logic, relational, frame, and production languages. Accounting for such an important element of automatic control system operation as real time operation and convenience in representation of information on the procedures and conditions for their application, the production model of knowledge description is the primary model used in fuzzy controllers in practice [148, 149].

Each production is represented as a set of rules which represent a knowledge fragment: the nucleus in knowledge engineering. The production takes the form "condition-action." The more complex version of a knowledge fragment "situation-control strategy-action" is examined in [9].

The left side of each production is treated as conjunctions of elementary (perpetual) conditions, while the right side is treated as a set of elementary actions. Any rule in the knowledge base for the controller shown in Fig. 1 can be represented as:

if (ε is ε_1^*) and ($\dot{\varepsilon}$ is $\dot{\varepsilon}_1^*$), then (u is u_1^*), where $\varepsilon, \dot{\varepsilon}, u$ are variables, while $\varepsilon_1^*, \dot{\varepsilon}_1^*, u_1^*$, are linguistic values.

The table lists sample linguistic rules that are knowledge bases of a fuzzy industrial controller for controlling a cracking fractionator (the operation of such a controller is examined in greater detail in [15]). The following designations of linguistic variables were used in this case: NL—negative large; NSL—negative sublarge; NM—negative mean; NSM—negative submean; NS—negative small; NSS—negative subsmall; NZ—negative zero; PZ—positive zero; PSS—positive subsmall; PS—positive small; PSM—positive subsmall; PM—positive mean; PSL—positive sublarge; PL—positive large.

The scale coefficients $K_\varepsilon, K_{\dot{\varepsilon}}, K_u$ are elements of the universal sets E, E', and U (in which the fuzzy sets $\varepsilon^*, \dot{\varepsilon}^*, u^*$ are represented) and are determined based on the conditions of the specific controlled object [15, 17].

The application of fuzzy set theory and knowledge bases to the design of fuzzy controllers makes it possible to improve their "intellect", competence, and to approach natural human intellect. The "humanization" of fuzzy controllers is one of the central problems in modern automatic control theory and engineering. We provide a brief survey of research in this field and note certain general aspects and differences between the cited research. Only the most representative studies are examined here, since this survey does not claim to provide comprehensive coverage of all publications in this field and is only illustrative in nature.[1]

2.2. **Analysis of industrial fuzzy controllers and automatic control systems.** One of the first studies devoted to the practical application of fuzzy logic controllers describes an experiment that carries out a heuristic synthesis of a controller for application to control of a steam turbine. A comparative analysis of the control results achieved using standard nonlinear digital devices and fuzzy controller demonstrated the advantage of the latter. An analogous study was carried out to assess the performance of a PI-controller and fuzzy logic controller used in a water heating control system. A number of common features identified in tests on fuzzy logic controllers should be noted.

—the authors have not relied on an exact model of the process in analyzing and designing fuzzy logic controllers. In this case a designer's intuition and his knowledge were incorporated directly in the control algorithm. This was followed by an iteration process to test algorithm performance, analyze its behavior and modify the corresponding control rules. Often this procedure requires significant time;

—fuzzy subsets included analogous linguistic variables such as PL (positive large) or NS (negative small), which reflects a general approach to choosing specific intervals of the quantized values;

—as a rule, all fuzzy logic controllers have employed the fundamental control principle of error-closing control [15–17].

In addition to the common aspects in the approach to the problem of constructing fuzzy logic controllers there are also differences in choosing the controller structure and in its application. One of the fundamental differences is in the problem of choosing a unique control action from the output fuzzy subset. One of the methods is to choose the control value for which there is a maximum membership function (when there are several points with a maximum membership function the mean for these points is chosen). This is the so-called mean of maximum method. The other method involves choosing the control value that represents the median of a figure bounded by the curve of the membership function of the output fuzzy subset. Such a method is called the center of area method. There are other varieties of analogous approaches such as the center of gravity method. Given the broad use of this method in the theory of fuzzy set design [83–85] additional research has been carried out; such studies have demonstrated the existence of more optimum fuzzy reasoning methods. Note that, when using the mean of maximum method, the fuzzy algorithm behaves as a positional relay, and methods of classical nonlinear automatic control systems can be used to analyze the fuzzy logic controllers. At the same time, when the center of area method is used, the fuzzy algorithm is identical to a PI-controller, and it is assumed that this method is preferred. An interesting element is the method of synthesizing fuzzy logic controllers based on a linguistic model of the object, which is demonstrated using a number of examples. However, the authors have been limited to examples for a first-order object without indicating the methods or possibility for extending this method to higher order objects, which significantly limits their application (see [15]).

Linguistic Rules Table (LRT)

		g^*						
		NL	NM	NS	NZ	PS	PM	PL
	NL	NZ	PS	PM	PL	PL	PL	PL
	NM	NS	NZ	PS	PM	PL	PL	PL
	NS	NM	NS	NZ	PS	PM	PL	PL
x^*	NZ	NL	NM	NS	NZ	PS	PM	PL
	PS	NL	NL	NM	NS	NZ	PS	PM
	PM	NL	NL	NL	NM	NS	NZ	PS
	PL	NL	NL	NL	NL	NM	NS	NZ
		e^*						

[1]These and other examples of the initial studies on the practical application of fuzzy logic controllers are described in [14–17, 38, 73–76, 140–153].

There are also differences in the problem of choosing inputs for the fuzzy logic controllers. Either errors and the rate of change of an error, or an error and the sum error, have been used as inputs for fuzzy logic controllers. An algorithm was discussed where the inputs were errors and the rate of change in the error, while the output was determined by two methods: if a high error value was present, the algorithm generated an absolute control, while in the case of a low error, the algorithm generated an incremental control action. Such an approach has been employed to optimize the response time of a system.

Note that the fuzzy logic controllers are required to correspond to traditional criteria in all cases, including transient process quality and decomposition of the control channels.

A broad range of minicomputers and mainframe computers and programming languages including FORTRAN, APL, BASIC, ASSEMBLER, C, etc. have been used in the development of FLC's.

Note also the several studies that have contributed to the development of the theory of fuzzy ACS models. The "observation operator" concentration has been examined; this can be used to define the state of a process more precisely. The idea of a "control goal" has also been proposed.

The problem in this case is formulated as follows. Let G be a fuzzy set in the state space X. Then the description of the processes includes: mapping in the state space f: $X \times U \to X$, and the observation operator Q: $x'Q = y$ and the mapping for the controller g: $Y \to U$.

Significant results have been obtained in developing the mathematical principles of fuzzy system analysis. The following mappings of f have been determined: f: $F(x) \times F(U) \to F(x)$; $F(x) \to F(y)$, where $F(x)$, $F(y)$ and $F(U)$ are the sets of all fuzzy subsets of state space X, input space U, and output space Y. The concepts of attainability, observability, and stability were introduced; these generalize the corresponding concepts of the fuzzy systems.

A theorem for approximation of linguistic algorithms of fuzzy systems by means of analytic functions has been formulated. This theorem provides the basis of analytic theory of fuzzy automatic control systems.

We also note the studies that have introduced arithmetic operations employing fuzzy sets and defined a fuzzy convolution integral and a fuzzy transfer function based on such sets. There is, however, an enormous excess information in the method, and there have been difficulties with its practical realization.

The problems of fuzzy system theory have also been discussed in [15, 66, 67]. Stability problems of fuzzy systems have been discussed and the principle of invariance has been developed for such systems.

Models of a fuzzy automatic control system for a nonlinear multidimensional process have been developed. The problem of decomposition of a control object (a steam turbine) so that the decomposed subsystems have asymptotic stability has been discussed. It was demonstrated from the modelling process that closed subsystems follow the desired state parameters under different operating conditions of the control object. The stability analysis employed a frequency criterion assuming that the decomposed systems are linear and that decomposition is correctly carried out. In conclusion, the study specifies the assumptions of which type of setting variation and controller structure are considered to be permissible.

Fuzzy logic has also been applied in self-learning controllers. It has been demonstrated that for many systems it is necessary to go from a purely descriptive approach to a prescriptive approach or to a self-organizing system. A drawback of fuzzy logic (as noted by the authors) is its explanatory form of knowledge representation, which does not permit using a prescriptive approach to decision making. It has been noted that fuzzy logic, like any other logic, can identify a consequence from pre-established premises. A prescriptive system is possible if a hierarchical approach to decision making is used; in this case the lower level strategy is determined based on an upper level description. A self-organizing controller utilizes this idea to identify control rules from an established desired system response and a preceding analysis of the behavior of the control object.

As noted in the literature [15–17] a generally promising direction in the development of FLC's is the development of adaptive and self-learning fuzzy systems. This is responsible for the appearance of many studies that are devoted to a discussion of the design of self-organizing controllers (SOC) [15–17, 154–159]. This type of controller has a hierarchical structure whose lower level contains the fuzzy controller itself, and whose upper level contains a monitor (corrector) that, when necessary, modifies the rules of the lower level controller. The monitor utilizes the desired response of the closed system which is represented as a correspondence table. This table has inputs that are identical to the inputs of the fuzzy logic controller (i.e., error and rate of error change), while the elements of the table demonstrate the degree to which the defined state of the system is assumed to deviate from the desired state. Therefore this correspondence table reflects the changes that must be introduced to the structure of the system by analyzing its behavior in the state space. The zero elements in the upper level table form the domain in the state space in which the FLC characteristic is a satisfactory characteristic. A continuous dependence of the outputs on the inputs is assumed for application of a self-organizing controller. If the system behavior deviates from the desired behavior, the upper level table will exert an action on the corresponding elements of the lower level table (hence the elements of the upper table are treated as gain). A parameter called the "gain delay" is also introduced. The iterative nature of the operation of a self-organizing controller is noted. Such a self-organizing controller was employed to control a steam boiler in two versions. Rules formulated by a human operator were used in the first case. In the second case the self-organizing controller contained no rules prior to the experiment. The results from both versions following an iterative self-learning procedure differed significantly.

This is due to the fact that the desired system response is not unambiguously determined, but rather lies in a certain domain in which it is possible to find a set of satisfactory solutions. Note that a self-organizing controller may operate with an initial set of rules that may even be inadequate rules. Self-organizing controllers are particularly significant in the control systems for backup equipment used to replace lost functions and artificial organs (such as artificial breathing devices, artificial kidneys, artificial circulation systems, etc.) [12, 13]. In this case, using fuzzy algorithms in combination with adaptation and self-organizing principles makes it possible to improve control processes under extremal situations and implement so-called benign conditions of backup equipment.

A detailed analysis of analytic methods of constructing fuzzy controllers to account for linguistic synthesis and adaptation problems is listed in [15–17].

Note also that the development of fuzzy processor hardware [9, 10, 15–30] has led to the development of a broad range of fuzzy controllers and automatic control systems that have found broad application in various fields of industry: robotic systems [160–174], spacecraft control [175], atomic reactors [176–180], petrochemical and gas industries [181–183], the mining industry [184–185], glass manufacturing [186], biological processes [187], underground transport (subway) control [188–192], cargo elevators [193–196], waste and trash processing and incinerating [87, 197, 198], control of ventilation in highway tunnels [199], cargo trains [200], and many other applications.

The experimental operation of multiple fuzzy automatic control systems has demonstrated satisfactory results.

3. Concluding Comments

The following should be noted in connection with this survey of research on fuzzy logic controllers and self-organizing controllers:

—fuzzy logic controllers can be used for controlling qualitative information that is difficult to formalize in realizing traditional control laws. In this case, fuzzy logic controllers are insensitive to perturbations within a given range and are characterized by enhanced performance compared to classical controllers [210–215];

—software has been developed for program-oriented fuzzy automatic control systems [216–221] to enhance the dynamical characteristics of the control processes;

—designer intuition and excellent knowledge of the object of control is required to write control laws for fuzzy logic controllers. However, the literature contains virtually no methods for direct synthesis of fuzzy logic controllers;

—altering the parameters of the object of the controller requires modification of the control rules of the fuzzy logic controller and their subsequent correction. Significant time is required to implement this procedure;

—as a rule, a self-organizing controller is used for initial synthesis of the lower level table. The studies examined here contain no information on the possible application of self-organizing controllers to industrial automatic control systems. One exception is a study where an automatic control system has been used to synthesize control rules for experimental control of a steam boiler;

—the possibility for the application of self-organizing controllers to a multidimensional process has not been examined in sufficient detail. This may be due to the fact that a secondary unit containing a flexible model of the object of control must be incorporated in the self-organizing controller for this purpose. As noted in [15], such a model is represented as a Jacobian, which presents additional difficulties for its calculation in the control process;

—there are no methodological aspects for formulating the upper hierarchical level of a self-organizing controller as a generator of the desired characteristic of the control system;

—there are insufficient studies (see [222–224, 154–158, 179]) that discuss the capability or methods of applying self-organizing controllers to the control of nonstationary multidimensional industrial facilities.

The authors of [15–17] have considered possible approaches to solving some of these problems as well as the current state of the problem of constructing and utilizing industrial fuzzy controllers and automatic control systems.

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