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Paper:

Computational Intelligence for Robust Control Algorithms of Complex Dynamic Systems with Minimum Entropy Production

Part 1: Simulation of Entropy- Like Dynamic Behavior and Lyapunov Stability

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Our thermodynamic approach to the study and design of robust optimal control processes in nonlinear (in general global unstable) dynamic systems used soft computing based on genetic algorithms with a fitness function as minimum entropy production. Control objects were nonlinear dynamic systems involving essentially nonlinear stochastic differential equations. An algorithm was developed for calculating entropy production rate in control object motion and in control systems. Part 1 discusses relation of the Lyapunov function (measure of stability) and the entropy production rate (physical measure of controllability). This relation was used to describe the following qualitative properties and important relations: dynamic stability motion (Lyapunov function), Lyapunov exponent and Kolmogorov-Sinai entropy, physical entropy production rates, and symmetries group representation in essentially nonlinear systems as coupled oscillator models. Results of computer simulation are presented for entropy-like dynamic behavior for typical benchmarks of dynamic systems such as Van der Pol, Duffing, and Holmes-Rand, and coupled oscillators. Parts 2 and 3 discuss the application of this approach to simulation of dynamic entropy-like behavior and optimal benchmark control as a 2-link manipulator in a robot for service use and nonlinear systems under stochastic excitation.

Keywords: Computational intelligence, Entropy production rate, Lyapunov stability, Entropy-like dynamic behavior

1. Introduction

The application of new knowledge-based control algorithms in advanced control theory of dynamic robotics systems has necessitated the development of new calculation such as computational intelligence (CI). Conventional basic computing tools for CI include genetic algorithms (GAs), fuzzy neural networks (FNNs), fuzzy set theory, evolution programming, and qualitative probabilistic reasoning. Application of CI to complex robotics motion control theory is divided into (1) the study of stable motion processes and (2) unstable motion processes of complex dynamic systems.

In the first case, stable motion, we describe intelligent control algorithm development and design (**Fig.1**). The feature of the given structure is the consideration of the control object based on fuzzy system theory as a black box, and the study and optimization of input-output linguistic relations using GA, FNN, and fuzzy control (FC) to describe the changing law of PID-controller parameters with minimum control error. In small uncontrollable (unobservable) external excitation or small parameter (or structure) change in control objects, such an approach ensures robust, stable control.

In a global unstable dynamic control object, such approach (a presence robust), does not guarantee stable control in principle. For such unstable dynamic control objects, we need a new intelligent robust algorithms based on knowledge about the movement of essentially nonlinear unstable

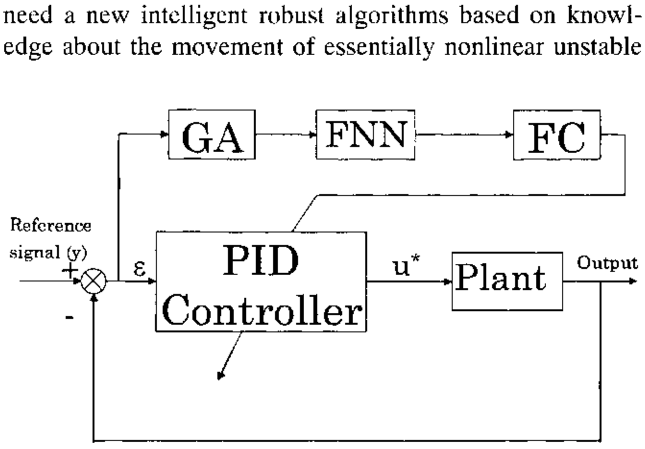

Fig. 1. AI Control (Designation in Text)

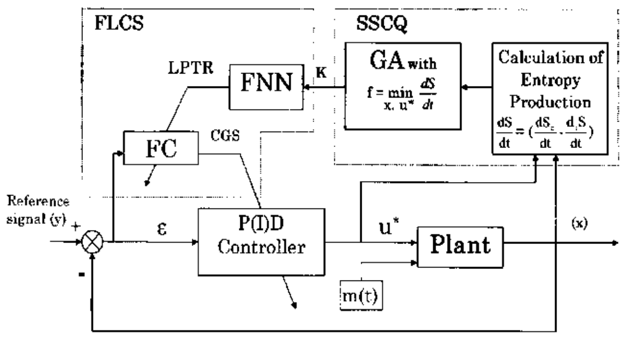

Fig. 2. Self-Organization AI Control with Physical Measure of Control Quality (Designation in Text)

dynamic systems. An example is the new benchmark, the robotic unicycle.[1] The general form of similar intelligent robust control algorithms is shown in **Fig.2**. Figures.1 and 2 use the following designations: GA: genetic algorithm; f: GA fitness function; S: system entropy; S_c: controller entropy; S_i: controlled plant entropy; ε: error; u^*: optimal control signal; m(t): disturbance; FC: fuzzy controller; FNN: fuzzy neural network; FLCS: fuzzy logic classifier system; SSCQ: simulation system of control quality; K: global optimum solution of coefficient gain schedule (teaching signal); LPTR: lookup table of fuzzy rules; CGS: coefficient gain schedule (k_1, k_2, k_3).

This approach was firstly presented[2] as a new physical measure of control quality for complex nonlinear controlled objects described as nonlinear dissipative models. This physical measure of control quality is based on the *physical law of minimum entropy production in intelligent control systems and in the dynamic behavior of complex control objects.* The problem of the minimum entropy production rate is *equivalent to* the problem of maximum released mechanical work as the optimal solution of corresponding Hamilton-Jacobi-Bellman equations. The variational fixed-end problem of maximum work *W* was shown to be equivalent to the variational fixed-end problem of minimum entropy production.[3] Both optimum solutions are equivalent for dynamic control of complex systems and the principle of minimum of entropy production guarantees the maximum released mechanical work with intelligent operations. This physical measure of control quality was used as a GA fitness function in optimal control design (Fig.2, Box SSCQ).

The introduction of physical criteria (minimum entropy production rate) guarantees the stability and robustness control of unstable objects. This differs from the aforesaid design (Fig.1) in that intelligent global feedback in control is used. The relation between control object stability (Lyapunov function) and controllability (entropy production rate) is used. The basic feature of this is the necessity of model study for control objects and the calculation of the entropy production rate through parameters of the developed model. The integration of joint systems of equations (equations of mechanical model motion and of the entropy production rate) enable the result to be used as the GA fitness function as CI. Part 1 describes a general approach to entropy definition and calculations from the dynamic system model movement equation and presents results of entropy-like dynamic behavior modeling of typical benchmarks of dynamic systems.

Parts 2 and 3 introduce a thermodynamic approach to studying and designing robust optimal control processes for nonlinear nonholonomic (in general global unstable) dynamic systems.

2. Definition of Entropy Production Rate and Relation to Lyapunov Stability in Nonlinear Closed Dissipative Dynamic Systems

One objective of thermodynamics is to characterize states of macroscopic systems depending on a limited number of observables. It is well established that large classes of dynamic systems present (under nonequilibrium conditions) complex behavior associated with bifurcation sometimes culminating in deterministic chaos. Much work has been done in characterizing this complexity. A variety of quantities related to the dynamic, including entropy-like, have been introduced and provide a rather successful description: Lyapunov exponents, Kolmogorov-Sinai entropy, and block entropies are representative examples. Our objective was to explore the possibility of introducing entropy production-like quantities related directly to the dynamics of complex systems and to assess their status for thermodynamic entropy production. This is achieved by adopting probabilistic formulation.[4]

Remark 1. In mechanics of continuous media, elasticity theory, and general dynamic system theory used methods and models described as irreversible in phenomenological thermodynamics.[5-9] Different approaches are used.[5-7,9] The phenomenological thermodynamic approach to correctness analysis of differential equations developed first[6,7] and necessary conditions for physical realization of differential equations as mathematical models for real dynamic systems were studied. The relation between the time rate of Lyapunov density and the time rate of excess availability dissipation in phenomenological thermodynamics[10] was then studied. The same problems from statistical thermodynamics[11-14] are discussed.

Remark 2. Relaxation processes were analyzed as a complex system describing compound parts of *mechanical* ⊕ *thermodynamic* behavior in dynamic systems from phenomenological thermodynamics[15]. Mechanical behavior of dynamic systems was described by the designated class of ordinary nonlinear differential equations. Thermodynamic behavior was characterized by entropy production and determined directly from mechanical system motion. The relation between entropy production rate and Lyapunov function for closed nonlinear relaxation processes in dynamic systems was introduced and its consequences discussed.[8,9,15]

Our purpose is to describe an application of the phenomenological thermodynamic approach[7] for analyzing any class of dynamic systems described by nonlinear dissipative differential equations. We studied relations between the notion of the Lyapunov function, entropy production rate, and the physical realization of approximate mathematical models describing irreversible processes in closed nonlinear dynamic systems.

Thermodynamic criteria (positive entropy production rate) as a physical measure for realizing a mathematical model (relaxation processes) are introduced. These criteria indicate the need to put extra (thermodynamic) limitations on parameters of differential equations and on qualitative properties describing dynamic evolution systems. We studied the correlation between conditions of physical realization and the notion of stability, and the correctness of mathematical models for irreversible processes in a nonlinear dissipative dynamic systems.

Such study is very important to correctly analyzing dynamic evolution and stability motion of dynamic systems,[7-9] and for describing artificial life conditions for micronanorobots.[15] Introducing a physical background in control processes is very important to designing optimal control processes using soft computing based on GA with a fitness function as *minimum entropy production* in the motion of a dynamic system and in the control process.[2]

2.1. Definition of Entropy-Like Dynamic Behavior of Complex Nonlinear Systems

Control objects are described based on classical mechanics using two approaches – Lagrangian and Hamiltonian equations.

For both approaches, we consider entropy production rate definition and calculation.

2.1.1. Lagrange's Approach

Consider Lagrangian equations

$$\frac{d}{dt}\left(\frac{\partial K}{\partial \dot{q}_i}\right) - \frac{\partial U}{\partial q_i} + \frac{\partial Q_i}{\partial \dot{q}_i} = F_i(t), \quad (1)$$

where $L = K - U$ is a Lagrangian of the dynamic system (1), $K = \frac{1}{2}\sum_{i,k=1}^{n} a_{ik}\dot{q}_i\dot{q}_k$ is kinetic energy, and $U = \frac{1}{2}\sum_{i,k=1}^{n} b_{ik}q_iq_k$ is potential energy of a dynamic system, q_i is a generalized coordinate.

In linear algebra, operator A is defined such that $q = A\xi$ or $q_i = A_{i1}\xi_1 + ... + A_{in}\xi_n$ $(i = 1, 2, ..., n)$ and $T = \frac{1}{2}\sum_{i=1}^{n} a'_i\,\dot{\xi}_i^2$; $U = \frac{1}{2}\sum_{j=1}^{n} b'_j\,\xi_j^2$. From Eq. (1) for a closed system, we obtain

$$\ddot{\xi}_i + f_i(\dot{\xi}_1, ..., \dot{\xi}_n) + \omega_i^2\xi_i = 0, \quad (i = 1, 2, ..., n). \quad (2)$$

Newton's Eq.(2) include additive nonconservative friction forces $f_i(\dot{\xi}_1, ..., \dot{\xi}_n)$.

Consider Lyapunov function V (for $a'_i = 1$ and $b'_j = \omega_j^2$) as full energy ($V \equiv E$)

$$V = \frac{1}{2}\sum_{i=1}^{n}\dot{\xi}_i^2 + \frac{1}{2}\sum_{i=1}^{n}\omega_i^2\xi_i^2 = T + U = E \quad (3)$$

and

$$\frac{dV}{dt} = \sum_{i=1}^{n}\dot{\xi}_i\ddot{\xi}_i + \sum_{i=1}^{n}\omega_i^2\xi_i\dot{\xi}_i. \quad (4)$$

After multiplication, Eq. (2) on $\dot{\xi}_i$ and summing index i from 1 to n we obtain the following equation:

$$\sum_{i=1}^{n}\dot{\xi}_i\ddot{\xi}_i + \sum_{i=1}^{n}\omega_i^2\xi_i\dot{\xi}_i = -\sum_{i=1}^{n}\dot{\xi}_i f_i(\dot{\xi}_1, ..., \dot{\xi}_n). \quad (5)$$

From Eqs. (4) and (5), it is follows that

$$\frac{dV}{dt} = -\sum_{i=1}^{n}\dot{\xi}_i f_i(\dot{\xi}_1, ..., \dot{\xi}_n) < 0. \quad (6)$$

The entropy production rate (for a closed system)

$$\frac{d_iS}{dt} = \frac{1}{T}\sum_{i=1}^{n}\dot{\xi}_i f_i(\dot{\xi}_1, ..., \dot{\xi}_n) > 0. \quad (7)$$

From Eqs.(6) and (7), we obtain

$$\frac{dV}{dt} = -\frac{1}{T}\frac{d_iS}{dt} < 0. \quad (8)$$

Thus, we obtain the general relation between the Lyapunov function (stability),[16] the entropy production rate, and the full energy of a dynamic system. This relation is a general one in the vibration theory of dynamic systems.

From Eq.(8), it follows that an infringement of thermodynamic criteria of physical realization on right side of Eq.(8) result in the instability of dynamic system and vice versa.

Example 1. Consider the dynamic system as

$$\ddot{q} + \phi(q) + \varphi(q, \dot{q}) = 0, \quad \varphi(0, \dot{q}) = \varphi(q, 0) = 0. \quad (9)$$

According to thermodynamic criteria (7)

$$\frac{1}{T}\frac{d_iS}{dt} = \varphi(q, \dot{q})\dot{q} > 0$$

For a particular case of Eq.(9)

$$\ddot{q} + \beta\dot{q} + \omega^2 q = 0, \quad \frac{1}{T}\frac{d_iS}{dt} = \beta\dot{q}\dot{q} = \beta\dot{q}^2$$

and coefficient β must be positive ($\beta > 0$).

For a dynamic system: $\ddot{q} + \beta\dot{q}^n q^m + \omega^2 q = 0$, $\frac{1}{T}\frac{d_iS}{dt} = \beta\dot{q}^{n+1}q^m$ and for $n = 1$, $m = 2$, $\beta > 0$, we have $\beta\dot{q}^2q^2 > 0$. For $m = 2i$ it is necessary that $n = 2i + 1$; $\beta > 0$, $(i - 1, 2, ..., l)$.

Similar relations exist between the entropy production rate and the correctness of dynamic systems.[7,8]

Example 2. *Entropy-Like Values and Criteria of Dynamic Accuracy in Nonlinear Automatic Control Systems (Accuracy of Linear Approximation).* Consider criteria of dynamic accuracy in a nonlinear automatic control system described as nonlinear equations[8]:

$$\dot{x}_1 = \sum_{j=1}^{n} a_{ij}x_j + \Phi(x_1) + f(t); \quad |f(t)| \le F_0$$

$$\dot{x}_1 = \sum_{j=1}^{n} a_{ij}x_j, \quad (i \ne 1) \quad \mathbf{(10)}$$

and a linear approximation as

$$\dot{x}_1 = \sum_{j=1}^{n} a_{ij}x_j + a_1x_1 + f(t); \ |f(t)| \le F_0, a_{10} \le a_1 \le a_{20}$$

$$\dot{x}_1 = \sum_{j=1}^{n} a_{ij}x_j, \ (i \neq 1). \quad (11)$$

Excitation $f(t)$ is a module-bounded function and for $f(t) = 0$ the system (11) is asymptotically stable. Necessary and sufficient conditions for linear approximation as Eq.(11) are described[8] as

$$\frac{dV_N}{dt} - \frac{dV_L}{dt} < 0, \quad (12)$$

where V_N and V_L are Lyapunov functions for nonlinear and linear systems corresponding. From the relation of Eq. (8) and Eq. (12) follows

$$\frac{d_iS_L}{dt} - \frac{d_iS_N}{dt} < 0. \quad (13)$$

Eq.(13) describing the entropy-like criteria of dynamic accuracy in linear approximation (46) of nonlinear automatic control (10) means that the entropy production rate in linear approximation must be less than in a corresponding nonlinear system.

This approach was used[7,8] to describe sensitivity and invariant conditions in nonlinear automatic dynamic control connected with the study of problems as excitation accumulations and stability on the part of generalized coordinates of dynamic systems.

2.1.2. Hamilton's Approach (Sympletic Geometry)

This approach unifies classical mechanics and thermodynamics. From mathematics, one postulate that the set of all states of a thermodynamic system is a differentiable manifold M and of the finite dimension. The evolution of the closed dynamic system is defined by semiflow $U: R_+ \times M \to M$ generated by vector field $Z(e) = \frac{d}{dt}U(t, e)|_{t=0}$, $e \in M$. The vector field does not depend explicitly on time, which means that the system is autonomous, or equivalently, is closed. The dynamic system must satisfy the two principles of thermodynamics. There exists energy state function $H: M \to R$, such that $Z \lrcorner dH = 0$, where $\lrcorner$ is the operator of the inner product. This only restates the conservation of energy principle. In the atlas of manifold M are coordinate systems in which state functions divide naturally in two classes: geometrical and nongeometrical state functions. To the first class belongs, for example, a state function such as the position or velocity of a point particle, and the second contains, for example, the energy or the temperature of the system. The essential difference between thermodynamic mechanical systems is that the latter is fully described without using nongeometrical state functions. The system is simple if, for each choice of special coordinate systems, the class of its nongeometrical state functions contains only one element.[17] We choose temperature as the unique nongeometrical state variable and study systems with positive temperature T, hence $M = \mathrm{M} \times R_+ = \{m, T\}$, $T > 0$. For simple systems, only one nongeometrical state variable is needed to describe all nongeometrical internal phenomena.

The second principle of thermodynamics is formulated as follows: if the system is adiabatically closed, there is nongeometrical state function $S: M \to R$, entropy, such that: 1) $\frac{\partial H}{\partial S} > 0$; and 2) $Z_t \lrcorner dS = \dot{S}_t \ge 0$. We not entirely recover work put into the system because part of it is always wasted by friction. The expression of S satisfying the third principle of thermodynamics ($S(m, 0) = 0$) $S(m, T) = \int_0^T \frac{1}{T}\frac{\partial H}{\partial t} dT$. Vector field $Z = (X, \dot{S})$ of a simple system is defined so that the two principles of thermodynamics are respected. We always use entropy as the nongeometrical state variable.

The first law for a closed simple system is reduced to $Z \lrcorner dH = X \lrcorner dH + T\dot{S}$. Hence $\dot{S} = -\frac{1}{T}X \lrcorner dH$ (see, Eq. (8)). The second law imposes $\dot{S} \ge 0$. To satisfy this inequality, we adopt Onsager's hypothesis by setting $\dot{S} = \frac{1}{T}\Lambda_s(X, X)$, where $\Lambda_s(X, X)$ is a positive semidefinite symmetric (s) quadratic form. Consequently, we have $X \lrcorner dH = -\Lambda_s(X, X)$. We postulate that field X is defined by $dH = -\Lambda(X)$, where $\Lambda: M \to (TM)_2^0$ is bilinear on TM. A vector field of $T_{(m,T)}M$ is given by a couple $X = (X_1, x_0)$, where $X_1 \in T_mM$ and $x_0 \in R$. [Decomposition $df = \mathbf{d}f + \frac{\partial f}{\partial T}dT$ is used and therefore $X \lrcorner df = X_1 \lrcorner df + \frac{\partial f}{dT}x_0$.] A bilinear form Λ always is decoupled in a sum of a symmetric antisymmetric form, i.e., $\Lambda = \Lambda_s + \Lambda_a$, and[17]

$$\dot{S} = \frac{1}{T}\Lambda(X, X) = \frac{1}{T}\Lambda_s(X, X) + \Lambda_a(X, X) = \frac{1}{T}\Lambda_s(X, X). \quad (14)$$

Equation $\dot{S} = \frac{1}{T}\Lambda_s(X, X)$ and $dH = -\Lambda(X)$ define completely vector field Z that correspond to the two principles.

In the field of sympletic mechanics, where state space is reduced to the submanifold, we know that vector field X is defined by $dH = -\Omega(X)$, where $\Omega(X)$ is a sympletic 2-form.[18,19] From this, the discussed model appears as a generalization of mechanics. The symmetric form, absent in mechanics, is introduced to account for dissipation. If the simple system is not isolated, vector field X_t is simply defined by $dh = -\Lambda(X) + \omega_t$, where ω_t is work defined by a differential 1-form as $\omega_t: R \times M \to T^*M$. Work ω_t is only produced by a change in system geometrical state variables; this leads us to impose an essential restriction on ω: $\forall x_0 \in R$, $(0, x_0) \lrcorner \omega = 0$. First principle $Z_t \lrcorner dH = Xt \lrcorner dH + T\dot{S} = X_t \lrcorner \omega_t + Q_t$ consequently gives $\dot{S} = \frac{1}{T}\Lambda_s(X, X) + \frac{1}{T}Q_t$. If $Q_t = 0$, i.e., if the system is adiabatically closed, then we recover $\dot{S} \ge 0$. Quantity $\frac{1}{T}\Lambda_s(X, X)$ is called the internal irreversibility of the system. A dissipative mechanical system is defined by manifold $M = T^*N$, where N is the configuration manifold.

Energy (Hamiltonian) and bilinear form Λ are given as[7,17]:

$$H(q, p, S) = K(p) + U(q, S);$$
$$\Lambda(q, p, S) = \Omega(p) + \Lambda_s(q, p, S), \quad (15)$$

where $K(p)$ is kinetic energy, $U(q, S)$ is internal energy (thermostatic energy) of the same system but in constrained equilibrium, $\Omega(p)$ is the symplectic form on T^*M,[17] and $\Lambda_s(q, p, S)$ is a positive semidefined symmetric bilinear form operating only on vectors of T^*N.

Example 3. Damped harmonic oscillator: $M = M \times R = R^2 \times R = \{(q, p), S\}$ with

$$H(q,p,S) = \frac{1}{2m}p^2 + \frac{1}{2}k(S)q^2 + f(S),$$
$$\Lambda(q,p,S) = dq \wedge dp + \Lambda_s(q,S)dqdp,$$

where $m > 0$ is mass, $k(S)$ is the spring constant, $f(S)$ is purely thermal energy and $\Lambda_s(q, S) \geq 0$ is interpreted as the friction coefficient. With notion $Z = (\dot{q}, \dot{p}, \dot{S})$, equation $dH = -\Lambda_s(X)$ gives

$$\frac{1}{m}pdp + k(S)qdq = -\dot{p}dq + \dot{q}dp - \Lambda_s(q,s)\dot{q}dq,$$

from which $\dot{q} = \frac{1}{m}p$, $\dot{p} = -\Lambda_s(q,s)\frac{1}{m}p - k(S)q$. The equation for the entropy production rate, $\dot{S} = \frac{1}{T}\Lambda_s(X, Y)$, is reduced to $\frac{d_iS}{dt} = \frac{1}{m^2T}\Lambda_s(q,S)p^2 \geq 0$ and equivalent to Eq.(7).

3. Relations Between Entropy Production Rate, Deterministic Chaos, and Lyapunov Function

From Eqs.(7) and (8), we can define a production entropy rate at the expense of irreversible processes through an accordingly selected Lyapunov function. The well-defined functions (wdfs) from a Lyapunov function is also known as Lyapunov functions.[16] Between these wdfs, we must choose the wdf that satisfies basic thermodynamic relations (8). The entropy production rate is a single-value function of dynamic system parameters and Eq. (8) lets us pick corresponding function V from the set of Lyapunov functions. For every differential asymptotic stable system (in Lyapunov sentence) there is a Lyapunov function[16] and from Eq.(8) it follows that a production entropy rate caused by an irreversible process in the system also exists.

3.1. Relations Between Entropy Production Rate, Deterministic Chaos, Lyapunov Exponent, and Kolmogorov-Sinai Entropy

Classical mechanics is said to be chaotic (or irregular) if adjacent trajectories in a given region of phase space diverge exponentially.[19] The largest Lyapunov number λ describes the asymptotic rate of exponential separation $d(t)$ between two initially close trajectories at distance $d(0)$: $\lambda = \lim_{d(0)\to 0, t\to\infty} \frac{1}{t}\ln\frac{d(t)}{d(0)}$. A universal quantity that is measured stochasticity is Kolmogorov-Sinai (KS) entropy h, defined in the general case of more than one region of connected stochasticity G[19]: $h = \int_G \left(\sum_{\lambda_+>0} \lambda_+(x)\right) d\mu$, where $d\mu$ is an element of the invariant volume (measure) in G, and x stands for conical coordinates and moments. In a single region of connected stochasticity xs are independent of x and the integral over $d\mu$ simply yields $h = \sum_{\lambda_+>0} \lambda_+$. In general, KS-entropy presents the mean of entropy production over the basin of an attractor. KS entropy h provides a measure of the rate of loss of information in predicting the future course of the trajectory and a dynamic system is said to be chaotic if h is positive definite.

For dynamic system $\dot{x} = F(x)$, where (x, F) are vectors in n-dimensional space R^n, in the neighborhood of given point x_0, we write $x = x_0 + \delta x$, where $\frac{d}{dt}\delta x = M(x_0)\delta x$, $M_{ij}(x_0) = -\frac{\partial F}{\partial x_j}\big|_{x=x_0}$. We construct orthogonal system $(e_0^1, e_0^2, ..., e_0^n)$ at point x_0. The *n-th* Lyapunov exponent in R^n is defined as follows: $\lim_{t\to\infty}\frac{1}{t}\log\left|\frac{e_t^1 \wedge e_t^2 \wedge \ldots \wedge e_t^n}{e_0^1 \wedge e_0^2 \wedge \ldots \wedge e_0^n}\right| = \sum_{i=1}^n \lambda_i$, where operation $\wedge$ presents a generalization of a vector product upon higher dimensional vector space. According to Liouville's theorem, $\frac{\partial}{\partial t}|e_t^1 \wedge e_t^2 \wedge ... \wedge e_t^n| = divF(x_t)|e_t^1 \wedge e_t^2 \wedge ... \wedge e_t^n|$.

Integration of this expression gives $\lim_{t\to\infty}\frac{1}{t}\log\left|\frac{e_t^1 \wedge e_t^2 \wedge \ldots \wedge e_t^n}{e_0^1 \wedge e_0^2 \wedge \ldots \wedge e_0^n}\right| = \lim_{t\to\infty}\frac{1}{t}\int_0^t divF(x_s)ds = \int\rho divfd\Gamma$ and, for a nonconservative dynamic system, the entropy production rate is[20] $\frac{dS}{dt} = \int\rho divFd\Gamma$, where density function ρ fulfills Liouville's equation. Thus $\frac{dS}{dt} = \int\rho divFd\Gamma = \sum_{i=1}^n \lambda_i = \sum_{i=1}^n \lambda_i^+ + \sum_{i=1}^n \lambda_i^-$, and for the entropy production rate due to irreversible process inside system $\frac{d_iS}{dt} = \sum_{i=1}^n \lambda_i^+$.

It is plausible that the rate at which information about the system is lost equals the average sum of positive Lyapunov exponents: $h = \int\rho\sum_{i=1}^n \lambda_i^+ d\mu$, where $\rho(x)$ is the invariant density of the attractor. Thus, $h = \int\rho\frac{d_iS}{dt}d\mu = \langle\frac{d_iS}{dt}\rangle$. In most cases, λ's are independent of x, so

$$h = \sum_{i=1}^n \lambda_i^+ \int\rho d\mu = \sum_{i=1}^n \lambda_i^+ \text{ and } h = \frac{d_iS}{dt}. \quad . . \ (16)$$

Remark 3. There is a close relationship between entropy production of nonequilibrium thermodynamics and *KS* entropy of dynamic system theory.[20] We have the entropy production rate due to irreversible processes inside the system (a role of bifurcation) $\frac{d_iS}{dt} = \sum_{k=1}^n \lambda_k^+$, where λ_k^+ is k-th positive Lyapunov exponent. For *KS* entropy with a differentiable map of a finite-dimensional manifold and an ergodic measure with a compact support Ruelle[19] it is shown that $h \le \sum_{j}^{n} \lambda_j^+$. According to Eq.(8), we write

$$\frac{d_i S}{dt} = -\frac{1}{T}\frac{dV}{dt} = \sum_{j=1}^{n} \lambda_i^+ \ge h. \qquad (17)$$

Eq. (17) is the generalized description of relations between the thermodynamic entropy production rate, dynamic stability of deterministic chaos, Lyapunov exponents, and *KS* entropy.

3.2. Statistical Definition of Entropy Production Rate and Symmetries of Stochastic Dynamics

The condition of instability of the two basic aspects, probability and irreversibility, are included, including chaos and nonintegrability in the sense of Poincare. There are also classes of situations where diffusive features play an essential role, e.g., situations studied in nonequilibrium statistical mechanics. For these situations, we must include in the fundamental description the two aspects so conspicuous on the macroscopic level, probability and irreversibility (consider bifurcation and chaos).[21] It is well established that large classes of dynamic systems present, under nonequilibrium conditions, complex behavior associated with bifurcations culminating in some cases to deterministic chaos. A natural description of complex dissipative systems should use the two principles of thermodynamics: 1) the principle of conservation of energy; and 2) the principle of nondecrease of entropy.

3.2.1. Statistical Definition of Entropy Production Rate

Having mapped dynamics stochastically, we inquire about properties of information (Shannon)-like entropies.[7,8] Specifically, one-time entropy is considered to be

$$S_I = -k_B\int \rho(x,t)\ln\rho(x,t)\,dx, \qquad (18)$$

where k_B is the Boltzman constant and $\rho(x, t)$ is the probability density function. The change of so-defined entropy, hence the time derivative of Eq. (18), then follows as

$$\frac{dS_I}{dt} = -k_B\int(1+\ln\rho(x,t))\left(\frac{\partial\rho(x,t)}{\partial t}\right)dx. \qquad (19)$$

According to Liouville's theorem for phase space $\frac{d\rho}{dt} = 0$ and

$$\frac{\partial\rho}{\partial t} = \sum_{i=1}^{3}\left(\frac{\partial\rho}{\partial\dot{x}_i}\frac{\partial H}{\partial x_i} - \frac{\partial\rho}{\partial x_i}\frac{\partial H}{\partial\dot{x}_i}\right) \qquad (20)$$

Inserting Eq.(20) in Eq.(19), we obtain $\frac{d\rho}{dt} = 0 \Rightarrow \frac{dS_I}{dt} = 0$, i.e., entropy change vanishes as long as Liouville's theorem applies to $\rho(x, \dot{x}, t)$. Liouville's theorem for phase space does not apply if, for example, particle-particle interactions and stochastically force a continuous time dynamic system to take place.

Example 4. The evolution of a stochastically forced dynamic system is given by a set of coupled first-order Langevin equations of the form

$$\dot{x} = F(x,u) + \xi(t), \qquad (21)$$

where x is the state vector, F the vector field, u a set of control parameters, and $\xi(t)$ stands for the effect of fluctuations of external noise on macroscopic dynamics. The effect is modeled as an additive multi-Gaussian white noise $\langle\xi_i(t)\xi_j(t')\rangle = D_{ij}\delta(t-t')$. The structure of covariance matrix D_{ij} (positive definite matrix) is imposed in external noise but follows from fluctuation-dissipative relationships in thermodynamic fluctuations. Eq.(21) defines a Markovian diffusion process and induces a Kolmogorov-Fokker-Planck (FPK) equation for the evolution of probability density function (PDF) $\rho(x, t)$ as[22]

$$\frac{\partial\rho}{\partial t} = -\sum\frac{\partial}{\partial x_i}(F_i\rho) + \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} D_{ij}\frac{\partial^2\rho}{\partial x_i\partial x_j}. \qquad (22)$$

Therefore, Eq. (19) is rewritten as

$$\frac{dS_I}{dt} = k_B\int(1+\ln\rho)\left\{\sum_i\frac{\partial}{\partial x_i}(F_i\rho) - \frac{1}{2}\sum_{i,j}\frac{\partial}{\partial x_i}\frac{\partial}{\partial x_j}(D_{ij}\rho)\right\}dx. \qquad (23)$$

Integrating the terms on the first sum twice by parts, we obtain

$$k_B\int(1+\ln\rho)\frac{\partial}{\partial x_i}(F_i\rho)\,dx = \int\rho\frac{\partial F_i}{\partial x_i}dx = \int\rho\, divF\,dx = \frac{d_e S_I}{dt}. \qquad (24)$$

The second term in Eq. (23) positive definite

$$\frac{d_i S_I}{dt} = \frac{1}{2}\sum_{i,j} D_{ij}\int\frac{1}{\rho}\left(\frac{\partial\rho}{\partial x_i}\right)\left(\frac{\partial\rho}{\partial x_j}\right)dx \ge 0 \qquad (25)$$

and presents the analogy of the Fisher information amount.

Thus, Eqs. (24) and (25) lead to identify the flux and (information) entropy rate correspondingly. Away from equilibrium, the entropy production rate (25) enters entropy balance through

$$\frac{dS_I}{dt} = \frac{d_e S_I}{dt} + \frac{d_i S_I}{dt} \qquad (26)$$

with $\frac{d_i S_I}{dt}$ as the entropy flow. In the absence of noise, expression (26) is reduced to the sum of Lyapunov exponents as in Eq.(17) – a negative quantity for a dissipative dynamic system.

Remark 4. For a particular case of μ-space ($x, y, z, p_x, p_y, p_z, t$), the FPK equation is as follows:

$$\frac{\partial\rho}{\partial t} = -\sum\frac{\partial}{\partial p_i}(F_i(p,t)\rho) + m^2\sum_{i-j}^{n}\sum_{j=1}^{n}\frac{\partial^2\rho}{\partial p_i\partial p_j}(D_{ij}(p,t)\rho),$$

where m is a mass of interacting particles, $F_i(p, t)$ drift vector components of the FPK equation, and $D_{ij}(p, t)$ diffusion tensor elements. Then

$$\frac{dS_I}{dt} = k_B \int (1 + \ln\rho) \left\{ \sum_i^n \frac{\partial}{\partial p_i}(F_i(p, t)\rho) - m^2 \sum_{i-j}^n \sum_{j=1}^n \frac{\partial^2 \rho}{\partial p_i \partial p_j}(D_{ij}(p, t)\rho) \right\} d\tau \quad . . (27)$$

Integrating in Eq. (27), terms of the first sum twice by parts

$$\int (1 + \rho) \frac{\partial}{\partial p_i}(F_i \rho)\, d\tau = \int \rho \frac{\partial F_i}{\partial p_i}\, d\tau .$$

Consider Maxwell-Boltzman nonisotropic distribution

$$\rho = g(x, y, z, t) \exp\left\{ -\frac{p_x^2}{2mk_BT} - \frac{p_y^2}{2mk_BT} - \frac{p_z^2}{2mk_BT} \right\}$$

with $g(x, y, z, t)$ as self-consistent charge density and the exponential function describing the distribution of the incoherent part of kinetic particle energy. The coherent part of kinetic energy is eliminated because it does not cause terms of the second sum of Eq. (27) evaluated to

$$m^2 \int (1 + \ln\rho) \frac{\partial^2 \rho}{\partial p_i \partial p_j}(D_{ij}(p, t)\rho)\, d\tau = -\frac{m}{k_BT}\delta_{ij} \int D_{ij}\rho\, d\tau.$$

In summary, the change of entropy caused by a Markovian process is expressed as an FPK-equation coefficient as[23)]

$$\frac{dS_I}{dt} = k_B \sum_{i=1}^n \left(\langle \frac{\partial F_i}{\partial p_i} \rangle + \frac{m}{k_BT} \langle D_{ij} \rangle \right), \quad . . . . (28)$$

where $\frac{d_eS_I}{dt} = k_B \sum_{i=1}^3 \langle \frac{\partial F_i}{\partial p_i} \rangle$ and $\frac{d_iS_I}{dt} = \sum_{i=1}^3 \frac{m}{T_i} \langle D_{ij} \rangle$.

Example 5. Consider the example of a simple yet nonlinear dynamic system in the presence of noise

$$\dot{x} = \gamma x - x^3 + \xi(t) . \quad . . . . (29)$$

In the noiseless limit and for $\gamma < 0$, system (29) admits a stable single fixed point whose Lyapunov exponent is $\lambda = \gamma$. This fixed point becomes repelling for $\gamma > 0$. In this range, two new simultaneously stable branches, $x = \pm\sqrt{\gamma}$, emerge from $x = 0$. Lyapunov exponents associated with these new attractors are $\lambda = -2\gamma$. In a weak noise limit and long time limit, Eqs. (24) and (25) are expressed[24)] as

$$\frac{d_eS_I}{dt} = \sum_{i=1}^n \lambda_i + 0(D_{ij}) , \quad . . . . (30)$$

$$\frac{d_iS_I}{dt} = -\sum_{i=1}^n \lambda_i + 0(D_{ij}) , \quad . . . . (31)$$

i.e., the flow and production of information entropy cancel each other.

Remark 5. Consider a simple system

$$\frac{dx}{dt} = p; \quad \frac{dp}{dt} = -\gamma p - \nabla U(x) + \xi(t) , \quad . . . . . (32)$$

where γ is the friction constant. Work done on the system by a reservoir is expressed as

$$(-\gamma p + \xi(t))\, dx = (\frac{dp}{dt} + \nabla U(x))\, dx = d(\frac{p^2}{2} + U(x)) .$$

In the large γ limit, we set $\frac{dp}{dt} = 0$, equivalent to $\gamma = 1$ to $dQ = dE$. Note that heat dQ absorbed by the system from the reservoir is given by $dQ = dE$, i.e., mechanical released work is equivalent to heat absorbed from the reservoir.[25)] This relation is used by calculation of the entropy production rate in complex dissipative mechanical systems.

3.2.2. Entropy Production Rate and Lie Symmetries of Nonlinear Dynamics

In the study of nonlinear dissipative dynamics, it is important to determine under what conditions a given dynamic system is integrable. Three techniques are widely used, namely Painleve analysis, Lie symmetries analysis, and a direct method of finding involutive integrals of motions. Among these, group theoretical methods are specifically significant. Given the nature of the symmetry vector field, one can also write integrals of motion for dynamic systems. The study of generalized Lie symmetries of nonlinear Hamiltonian systems gives integrable parameters and integrals of motion, and also separable coordinates if they exist. The existence of symmetry for differential equations leads to a reduced order in ordinary differential equations or difference equations and to a particular solution in partial differential equations.

For the set of first-order coupled nonlinear ordinary differential equations of motion as $F_i(x_j, \dot{x}_j) = 0$, $i = 1, 2, ..., N$; $j = 1, 2, ..., M$, invariance study under one-parameter infinitesimal point transformations of the form

$$X_i = x_i + \varepsilon\eta_j(t, x_i) , \qquad \mathrm{T} = t + \varepsilon\xi(t, x_i) , \quad i = 1, 2, ..., M \quad . . . . (33)$$

The corresponding infinitesimal generator is

$$V = \xi(t, x_i)\frac{\partial}{\partial t} + \eta(t, x_i)\frac{\partial}{\partial x_i} , \quad . . . . (34)$$

We take $\xi = 0$ without loss of generality. The evolutionary vector field takes one form

$$V = \eta_i(t, x_i)\frac{\partial}{\partial x_i} . \quad . . . . (35)$$

For the study of Lie symmetries of the set of coupled ordinary differential equations of the first order, we must know first prolongation $P_r^{(1)}$ of vector field V.[26)]

The associated first extended operator is

$$P_r^{(1)}V = \eta_i\frac{\partial}{\partial x_i} + \dot{\eta}_i\frac{\partial}{\partial \dot{x}_i} , \quad . . . . (36)$$

where $\dot{\eta}_i = D_t\eta$, $i = 1, 2$, and D_t is the total differential operator of a one-parameter symmetry group[26)] for system $F_i(x_j, \dot{x}_j) = 0$, whenever Eq.(36) is satisfied

$$P_r^{(1)}V(F_i)|_{F_i=0} = [\eta_i\frac{\partial}{\partial x_i} + \dot{\eta}_i\frac{\partial}{\partial \dot{x}_i}](F_i) = 0 . . . . (37)$$

Substituting the specific equation of motion $F_i = 0$ in Eq.(37) and solving it consistently, we get Lie symmetries η_i.

Remark 6. The second and higher prolongation of vector field V does not lead to nonsymmetries for the equation of motion. For example, the second prolongation

$$P_r^{(2)}V = \eta_i \frac{\partial}{\partial x_i} + \dot{\eta}_i \frac{\partial}{\partial \dot{x}_i} + \ddot{\eta}_i \frac{\partial}{\partial \ddot{x}_i}\,, \quad \ddot{\eta}_i = \frac{d^2\eta_i}{dt^2}\,,$$
$$i = 1, 2, \ldots .$$

Acting on system $F_i = 0$ leads to invariance conditions

$$P_r^{(2)}V(F_i)|_{F_i=0} = [\eta_i \frac{\partial}{\partial x_i} + \dot{\eta}_i \frac{\partial}{\partial \dot{x}_i}](F_i) = 0\,,$$

the same as the right side of Eq.(37). A similar results holds for higher prolongations.[26)]

Example 6: Holmes-Rand Nonlinear Oscillator.

Consider an application of stability analysis based on the relation between entropy production and the Lyapunov function to a benchmark as a Holmes-Rand oscillator

$$\ddot{x} + (\alpha + \beta x^2)\dot{x} - \gamma x + x^3 = 0\,, \quad \ldots\ldots\ldots\ (38)$$

where α, β, and γ are parameters. System (38) closely resembles the Duffing-van der Pol class of nonlinear oscillators. Rewriting Eq.(38) into a set of two first-order equations, we get

$$\dot{x} = y\,,\ \dot{y} = -(\alpha + \beta x^2)y + \gamma x - x^3\,. \quad \ldots\ldots\ (39)$$

The invariance requirement under infinitesimal transformation (37) is written as

$$\dot{\eta}_1 = \eta_2\,,\ \dot{\eta}_2 = (-2\beta xy + \gamma - 3x^2) - \eta_2(\alpha + \beta x^2)\,.$$

An ansatz for η_1 and η_2 are polynomials in variable y to have a nontrivial set of Lie vector fields:

$$\eta_1 = a_1 + a_2 y + a_3 y^2\,;\ \eta_2 = b_1 + b_2 y + b_3 y^2\,,$$

where a_i and b_i, i = 1, 2, 3, are functions of t and x alone.

We get a four-parameter symmetry group[26)] where associated vector fields and the dynamic vector field

$$X = y\frac{\partial}{\partial x} - \left[\left(\beta x^2 + \frac{4}{\beta}\right)y + x^3 + \frac{3}{\beta^2}x\right]\frac{\partial}{\partial y} \quad \ldots\ldots\ (40)$$

are associated with Eq.(38). From Eq.(40), it follows that dynamic system (38) has specific symmetries for $\alpha = \frac{4}{\beta}$ and $\gamma = -\frac{3}{\beta^2}$. Integral of motion I for Eq.(38) for choice $\alpha = \frac{4}{\beta}$ and $\alpha = -\frac{3}{\beta^2}$ is

$$I = \exp[(3/\beta)t]\,[\dot{x} + \frac{\beta}{3}x^3 + \frac{1}{\beta}x] \quad \ldots\ldots\ldots\ (41)$$

and we end up with a first-order inhomogeneous Abel's equation:

$$\dot{x} + \frac{1}{3}\beta x^3 + \frac{1}{\beta}x = I\exp[-(\frac{3}{\beta})t]\,. \quad \ldots\ldots\ldots\ (42)$$

The Holmes-Rand nonlinear oscillator does not pass the Painleve' test because it admits a movable algebraic branch point. It has been pointed out that there exist second-order systems that are non-Painlevean but nevertheless possess one integral of motion and hence are integrable. Choice $\alpha = \frac{4}{\beta}$ and $\gamma = -\frac{3}{\beta^2}$ of the Holmes-Rand nonlinear oscillator belongs to the above category.[26)]

The Lyapunov function for system (38) is described as $V = \frac{1}{2}\dot{x}^2 + U(x)$, where $U = \frac{1}{4}x^4 - \frac{1}{2}\gamma x^2$. The entropy production rate in system motion is $\frac{d_iS}{dt} = (\alpha + \beta x^2)\dot{x}^2$. Eq.(38) is written as $\ddot{x} + (\alpha + \beta x^2)\dot{x} + \frac{\partial U}{\partial x} = 0$ and, after multiplying the left part of this equation on $\dot{x}$, we obtain $(\ddot{x} + (\alpha + \beta x^2)\dot{x} + \frac{\partial U}{\partial x})\dot{x} = 0$. The value $\frac{dV}{dt}$ calculated as $\frac{dV}{dt} = \dot{x}\ddot{x} + \frac{\partial U}{\partial x}\dot{x}$ and after a simple algebraic transformation we obtain

$$\frac{dV}{dt} = -\frac{1}{T}\frac{d_iS}{dt}\,, \quad \ldots\ldots\ldots\ldots\ldots\ (43)$$

where T is a normalization factor.

An analysis of relations in Eqs.(41)-(43) shows that specific symmetries of Eq.(38) for $\alpha = \frac{4}{\beta}$ and $\gamma = -\frac{3}{\beta^2}$ with dynamic vector field (40) produce maximum stable non-equilibrium states with minimum production entropy.[2)]

For $\beta = 0$ from Eq. (38), we obtain the equation of motion of the force-free Duffing oscillator and for exactly the same parametric choice $2\alpha^2 = 9\gamma$ the Panlevean property holds[2,26)] for this equation. In this case[26)] integral of motion $I = \left(\exp\left\{\frac{4}{3}\alpha t\right\}\right)[\dot{x}^2 + \frac{2}{3}\alpha x\dot{x} + \frac{1}{2}x^4 + \frac{2}{9}\alpha^2 x^2]$. The relation in Eq.(43) is true with the Lyapunov function as for the Holmes-Rand oscillator and $\frac{d_iS}{dt} = \alpha\dot{x}^2$.

Eqs.(8) and (43) describe a generalized relation between Lyapunov functions V (qualitative measure of mechanical motion) and an entropy production $\frac{d_iS}{dt}$ (quantitative measure of thermodynamic behavior). For $\alpha = -1$, $\beta = 1$, $\gamma = -1$ and without nonlinear term x^3, we obtain the result for the Van der Pol oscillator.[27)]

3.2.3. Entropy Production Rate and Symmetries of Stochastic Dynamics

For the n-dimensional dynamic system admitting an n-linearity independent Lie symmetry vector field, the probability density function is found analytically in terms of these symmetries. For dynamic systems with a vector field having constant divergence and a first integral, the probability density function is written in term of these. The entropy production rate is calculated analytically for these systems.[28)] In general, the form for dynamic system $\dot{x}^i = f^i(x, t)(i = 1, 2, \ldots, n)$, $x \in G$ and associated probability density function $\rho(x, t)$. Liouville's theorem is

$$(\frac{\partial}{\partial t} + L_F)\,\rho(x, t)\Omega = 0\,, \quad \ldots\ldots\ldots\ldots\ (44)$$

where $\Omega = \sqrt{|g|}dx^1 \wedge dx^2 \wedge \ldots \wedge dx^n$, $g = \det(g_{ij})$ and $\wedge$ stands for the exterior product, L_F is the Lie derivative, $F = f^i(x,$

$t)\frac{\partial}{\partial x^i}$. This leads to a partial differential equation

$$\frac{\partial \rho}{\partial t} + f^m \frac{\partial \rho}{\partial x^m} + \rho divF = 0 \quad (m = 1, 2, ..., n) \quad (45)$$

Dynamic system (44) is invariant under the Lie group by the symmetry vector field (infinitesimal generator):

$$X^i = \eta^{i'} \frac{\partial}{\partial x^i} + \tau(x, t)\frac{\partial}{\partial t} \quad (i = 1, 2, ..., n), \quad (46)$$

if infinitesimal $\eta^{i'}(x, t)$ and $\tau(x, t)$ satisfy the system of partial differential equations

$$\frac{\partial \eta^{i'}}{\partial t} = L_F \eta^i, \quad \eta^i(x, t) = \eta^{i'}(x, t) - \tau(x, t) f^i(x, t). \quad (47)$$

Vector field $X = \eta^i(x, t)\frac{\partial}{\partial x^i}$ is called evolutionary representation[28] of the symmetry vector field given in Eq.(46). If the dynamic system given by (44) admits an n-linearly independent symmetry vector field having evolutionary representation of form $X_1 = \eta_1^{i_1}(x, t), ..., X_n = \eta_n^{i_n}(x, t)\frac{\partial}{\partial x^{i_n}}$; $(i_1, ..., i_n = 1, 2, ..., n)$ then Liouville measure $\mu(G) = \int_{G \subset \Gamma} \rho(x, t)\Omega$ is written with probability density function $\rho(x, t) = \frac{1}{|X_1 \rfloor X_2 \rfloor \dots \rfloor X_n \Omega|}$ and n-linearly independent divergence-free symmetry vector field X_i enjoys commutator relations[28]

$$[X_j, X_k] = \sum_{j<l<k} C_{jk}^l X_l \quad (j, k, l = 0, 1, 2, ..., n-1) \quad (48)$$

where C_{jk}^l are structure constants. If one of the first integrals for system (44) is known and given by $I(x, t)$, then $\rho(x, t) = \frac{|I(x, t)|}{|X_1 \rfloor X_2 \rfloor \dots \rfloor X_n \Omega|}$ is also a probability density function. If the vector field has constant divergence and a first integral, the PDF is written as $\rho(x, t) = |I(x, t)| \exp\{-\alpha t\}$, where $\alpha = \frac{\partial f^m}{\partial x^m}$ is a constant.

The entropy production rate is written

$$\dot{S} = \frac{dS}{dt} = -\int_{G \subset \Gamma} \left(\frac{\partial}{\partial t} + L_F\right) \rho \ln \rho \Omega = \int_{G \subset \Gamma} \rho \frac{\partial f^m}{\partial x^m}. \quad (49)$$

The entropy production rate in symmetries and first integrals is expressed as

$$\frac{dS}{dt} = \int_{G \subset \Gamma} \frac{1}{| X_1 \rfloor X_2 \rfloor \dots \rfloor X_n \Omega |} \frac{\partial f^m}{\partial x^m} \Omega \text{ and}$$

$$\frac{dS}{dt} = \int_{G \subset \Gamma} \frac{|I(x, t)|}{| X_1 \rfloor X_2 \rfloor \dots \rfloor X_n \Omega |} \frac{\partial f^m}{\partial x^m} \Omega.$$

Consider a general case of a stochastic dynamic Stratonovich system defined on some domain U in Euclidian space R^n as

$$dx_t = b(x_t, t)\, dt + \sum_{r=1}^{m} g_r(x_t, t) \circ dw_t^r,$$
$$x_{t_0} = c, \quad t \in I = [t_0, T) \quad (50)$$

where $w_t = (w_t^r)_{r=1}^m$ is an m-dimensional standard Wiener process. Ref.29) defined differential operators on U by $\partial_t = \frac{\partial}{\partial t}$, $X_0 = \sum_{i=1}^{n} b^i \frac{\partial}{\partial x_i}$, $X_r = \sum_{i=1}^{n} g_r^i \frac{\partial}{\partial x_i}$. It also defined function $y = \phi(x, t)$ as a transformation from $U \times I \to U$. Function ϕ is a symmetry transformation for stochastic dynamic system (50) if the function satisfies

$$b(\phi(x, t), t) = (\partial_t + X_0)\phi(x, t),$$
$$g_r(\phi(x, t), t) = X_r \phi(x, t) \quad (51)$$

For stochastic differential Stratonovich equations and Eq.(51), we have the following differential equation describing process $y_t = \phi(x_t, t)$ as

$$dy_t = (\partial_t + X_0)\phi(x, t)\, dt + \sum_{r=1}^{m} X_r \phi(x_t, t) \circ dw_t^r$$
$$= b(y_t, t)\, dt + \sum_{r=1}^{m} g_r(y_t, t) \circ dw_t^r, \quad (52)$$

where x_t is a diffusion process governed by Eq.(50). This means a stochastic dynamic system described by Eq.(44) is invariant under transformation satisfying Eq. (51). Such a transformation is called *a symmetry*.

If $y = \phi(x, t, a)$ is a local one-parameter transformation generated by differential operator $Y = \sum_{i=1}^{n} f^i(x, t)\frac{\partial}{\partial x^i}$ on U, $f = (f^i)|_{i=1}^n$ is an R^n-valued smooth function, a is a parameter on $J = (-a_0, a_0)$, $\phi(x, y, 0) = x$ then one-parameter transformation y is a symmetry transformation of stochastic dynamic system (50), if operator Y satisfies

$$[\partial_t + X_0, Y] = 0, \quad [X_r, Y] = 0 \quad (r = 1, 2, ..., m), \quad (53)$$

where $[\cdot,\cdot]$ denotes a Lie bracket. Eqs. (53) are equivalent to

$$(\partial_t + X_0) f^i = \sum_{j=1}^{n} f^i \frac{\partial}{\partial x^i} b^i, \quad X_r f^i = \sum_{j=1}^{n} f^i \frac{\partial}{\partial x^j} g_r^i$$
$$(i = \overline{1, n}, \; r = \overline{1, m}) \quad (54)$$

Operator Y satisfying (53) and (54) is a symmetry operator of Eq.(44) and has similar properties for description of the entropy production rate in symmetries vector fields.

4. Definitions and Simulation Results of Entropy-Like Behavior of Benchmarks as Typical Dynamic Systems

Box SSCQ in self-organized AI control (Fig.2) for calculating the entropy production rate in control objects and in control systems. We cite as an example benchmarks of entropy production calculation for coupled nonlinear oscillator models described by ordinary nonlinear differential equations of motion.

4.1. Definition of Entropy Production Rates of Benchmarks

The thermodynamic model representation of dynamic equations of motion for a control object (Plant) in a general form as closed and open dynamic systems is developed in Ref.2). Based on this, the analysis on Plant's postural stability control is done and results of computer simulation is compared.

Let us introduce results of entropy production calculation and dynamic behavior for typical systems as

1. *Van der Poll Oscillator Model*

$$\ddot{x} + (x^2 - 1)\dot{x} + x = 0$$

Entropy Production

$$\frac{dS}{dt} = \frac{1}{T}(x^2 - 1)\dot{x}^2$$

2. *Duffing Oscillator Model*

$$\ddot{x} + \dot{x} - x + x^3 = 0$$

Entropy Production

$$\frac{dS}{dt} = \frac{1}{T}\dot{x}^2$$

3. *Holmes-Rand (Duffing-Van der Pol) Oscillator Model*

$$\ddot{x} + (x^2 - 1)\dot{x} - x + x^3 = 0$$

Entropy Production

$$\frac{dS}{dt} = \frac{1}{T}(x^2 - 1)\dot{x}^2 .$$

4. *Duffing Oscillator Model with parametric excitation of dissipative force*

$$\ddot{x} + k(1 + A\sin\omega t)\dot{x} - x + x^3 = 0$$

Entropy Production

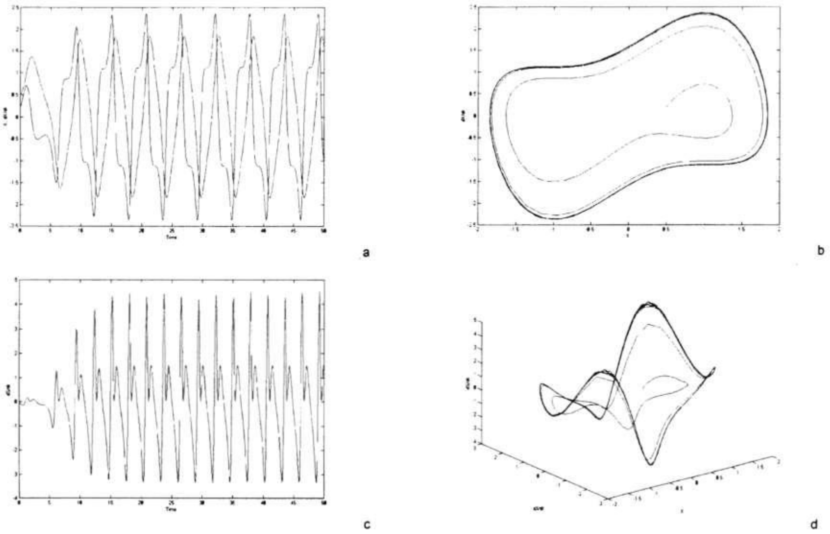

Fig. 3. Simulation results of entropy-like behavior for Holmes-Rand oscillator. *a:* Free motion with initial states $x_0 = 0.5$; $\dot{x}_0 = 0.2$; *b:* phase portrait; *c:* temporal entropy-like behavior (entropy production rate); *d:* 3D simulation entropy-like behavior.

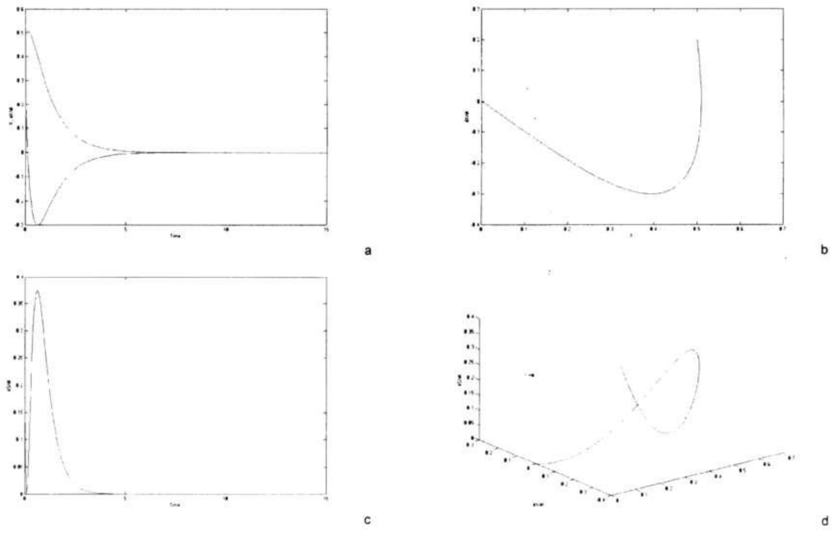

Fig. 4. Simulation results of entropy-like behavior for Holmes-Rand oscillator with symmetrics - $\alpha = 4/\beta$, $\gamma = -4/\beta^2$: *a:* free motion with initial states $x_0 = 0.5$; $\dot{x}_0 = 0.2$; *b:* phase portrait; *c:* temporal entropy-like behavior (entropy production rate); *d:* 3D simulation entropy-like behavior.

$$\frac{dS}{dt} = \frac{k}{T}(1 + A\sin\omega t)\dot{x}^2$$

Also consider the Lyapunov function as

$$V = \left(\frac{\dot{x}^2}{2} + \frac{\dot{y}^2}{2} + \frac{x^2}{2} + \frac{y^2}{2} + \frac{\alpha(x-y)^2}{2}\right) > 0, \quad \ldots \quad (55)$$

In this case,

$$\frac{dV}{dt} = \dot{x}\ddot{x} + \dot{y}\ddot{y} + x\dot{x} + y\dot{y} + \alpha(x\dot{x} - \dot{x}y - \dot{y}x + y\dot{y}), \quad (56)$$

The dynamic system with Lyapunov function (55) is a system of two coupled nonlinear oscillators as

$$\begin{cases} \ddot{x} + (\dot{x}^2 + x^2 - 1)\dot{x} + x + \alpha(x-y) = 0 \\ \ddot{y} + (\dot{y}^2 + y^2 - 1)\dot{y} + y + \alpha(y-x) = 0 \end{cases} \quad \ldots \quad (57)$$

After multiplication on $\dot{x}$ and $\dot{y}$ in both equations of system (57) and after simple algebraic transformations, we write final results as follows:

$$\frac{dV}{dt} + (\dot{x}^2 + x^2 - 1)\dot{x}^2 + (\dot{y}^2 + y^2 - 1)\dot{y}^2 = 0. \quad \ldots \quad (58)$$

From Eq.(58), it follows

$$\frac{dV}{dt} = -\sum_{i=1}^{2} \frac{d_i S}{dt}. \quad \ldots \quad (59)$$

From Eq.(59), we get the law on additive properties of entropy production in dynamic systems.

In a more complex case as

$$V = \left(\frac{\dot{x}^2}{2} + \frac{\dot{y}^2}{2} + \frac{\dot{z}^2}{2} + \frac{x^2}{2} + \frac{y^2}{2} + \frac{z^2}{2} + \frac{\alpha([x-y]-z)^2}{2}\right),$$

we have

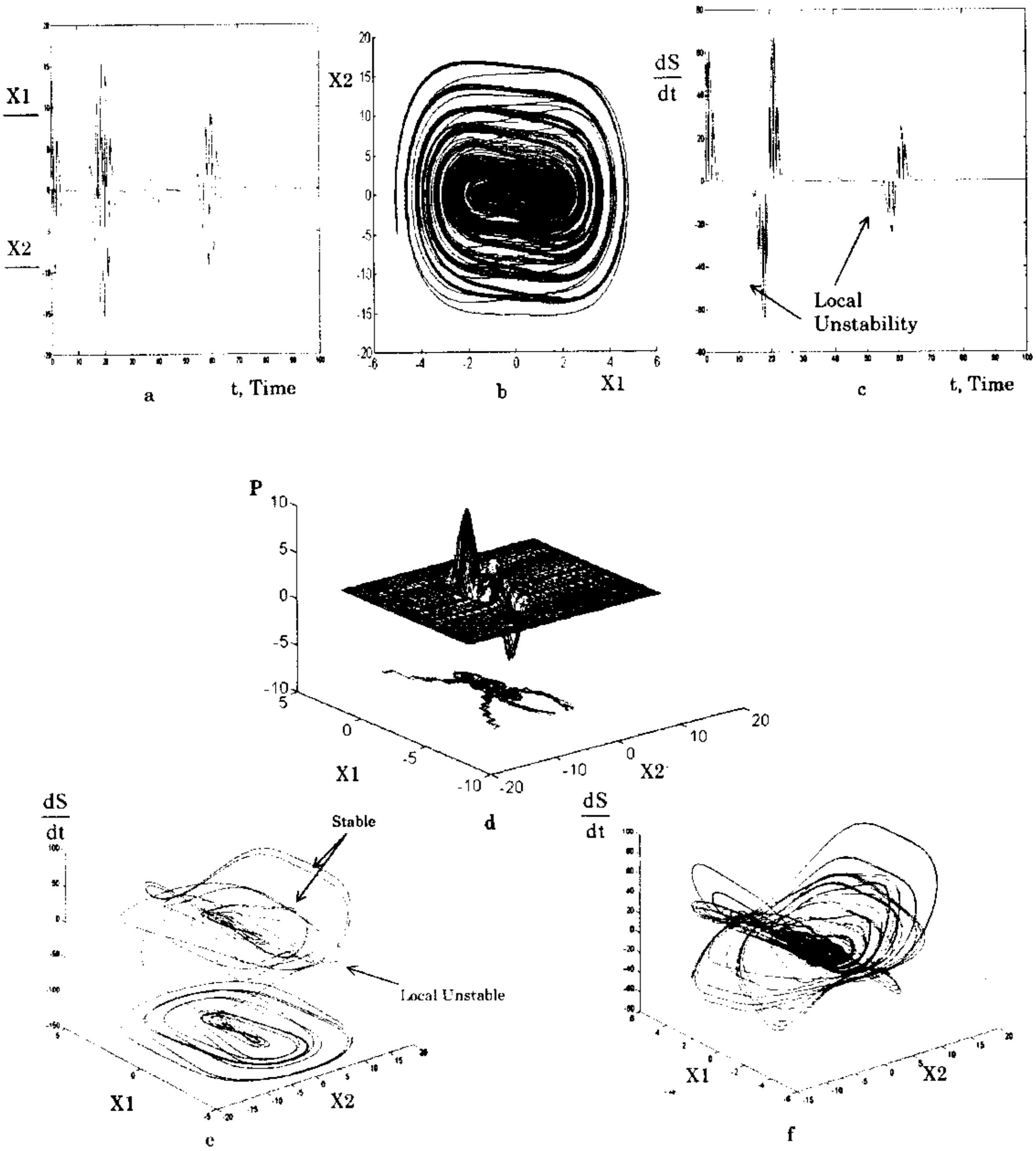

Fig. 5. Entropy analysis of stability of nonlinear parametric dissipative Duffing oscillator: a: temporal behavior of oscillator with initial states $x_0 = -1.75$, $\dot{x}_0 = -1$; A = 5; $\omega = 0.1$; *b:* the phase portrait; *c:* temporal entropy-like behavior with local unstable states; *d:* power distribution; *e:* 3D simulation of entropy-like behavior with one unstable state; *f:* 3D simulation of entropy-like behavior with *n* unstable states.

$$\begin{cases} \ddot{x} + (\dot{x}^2 + x^2 - 1)\dot{x} + x + \alpha(x + y - z) = 0 \\ \ddot{y} + (\dot{y}^2 + y^2 - 1)\dot{y} + y + \alpha(y - x + z) = 0 . \\ \ddot{z} + (\dot{z}^2 + z^2 - 1)\dot{z} + z + \alpha(z - x + y) = 0 \end{cases}$$

and $\frac{dV}{dt} = -\sum_{i=1}^{3} \frac{1}{T_i}\frac{d_iS}{dt}$; $\frac{d_1S}{dt} = \frac{1}{T_1}(\dot{x}^2 + x^2 - 1)\dot{x}^2$; $\frac{d_2S}{dt} = \frac{1}{T_2}(\dot{y}^2 + y^2 - 1)\dot{y}^2$; $\frac{d_3S}{dt} = \frac{1}{T_3}(\dot{z}^2 + z^2 - 1)\dot{z}^2$, for three coupled nonlinear oscillators.

4.2. Simulation of Entropy-Like Behavior of Complex Dynamic Systems.

- *Dynamic systems with one DOF: Consider* the benchmark of a dynamic system as a Van der Pol oscillator. The result of integration of a differential equation for mechanical motion together with the equation for the entropy production rate is shown in **Fig.3**, where the system is in local unstable equilibrium states. **Figure 4** shows the result of Holmes-Rand oscillator behavior with symmetries. In Fig.4c, the oscillator with symmetries ($\alpha = \frac{4}{\beta}$, $\gamma = -\frac{3}{\beta^2}$) produces less entropy according to symmetries analysis of the Holmes-Rand oscillator. **Figure 5** shows simulation results of entropy-like behavior in dynamic systems with parametric excitation. Periodic solutions and bifurcation behavior in a parametrically damped Duffing equation in terms of Floquet theory were studied.[30] At the stability boundary of solutions $x = \pm 1$, $\dot{x} = 0$, a common feature is that one of the eigenvalues of stationary solutions is always -1 and bifurcation is rather rich along the boundary. Dynamics differ and are interesting as parameters are valued to cross different seg-

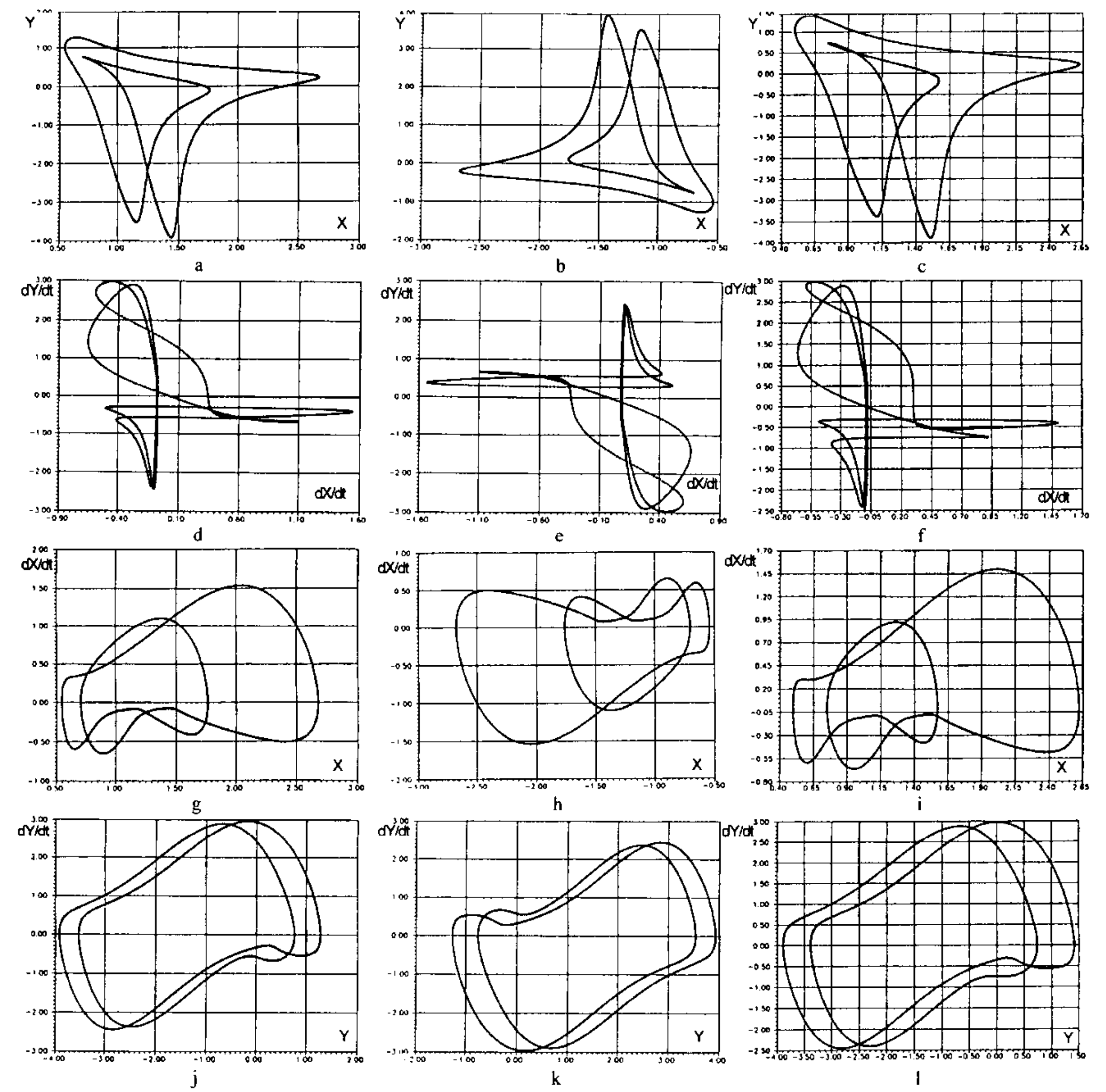

Fig. 6. Temporal behavior of mechanical motion of two coupled van der Pol oscillators: *a-c:* phase portraits (x, y); *d-f:* velocity phase portraits $(\dot{x}, \dot{y})$; *g-l:* phase portraits $(x, \dot{x})$; *j-l:* phase portraits $(y, \dot{y})$.

ments of this boundary. Numerical results shows that a period-two solution arises via period-doubling bifurcations at A = 5.385. The regions of various types of motions heavily overlap, indicating the coexistence of multiple attractors. In overlapped regions, at one parameter point, the system is attracted in different orbits, depending on initial preparation. At parameters $\omega = 1.45$ and $A = 5.38 \div 8.5$, there are five attractors. In addition to the pair of stationary solutions, there exists a pair of single-well period-one solutions and a symmetric period-two solution. Boundaries of basins of coexisting attractors clearly form fractals.[30] As $2.1 \le \omega \le 2.49$, system motion after instability is be larger-scale cross-well chaos via intermittency; at the critical point, one eigenvalue of stationary solutions is still −1. However, as $\omega \le 2.1$, system behavior becomes period-double bifurcation after stationary solutions lose stability (Fig.5). Strange attractors produce the stationary entropy production rate and a change of the frequency of parametric excitation brings up local instability. A similar effect is described for equations as

$$\ddot{x} + (\alpha_1 \sin\omega t)\dot{x} + \alpha_2\dot{x}^3 = \frac{1}{(\cos\Omega t - x)^2},$$

where $\omega = 0.04$ and $\Omega = \sqrt{2}$[31] for dynamic path planning of a mobile robot in nonstationary obstacle environments.

- *The dynamic systems of two-coupled nonlinear oscillators:* A system of two-coupled Van der Pol oscillators showing multistable behavior for some control parameters is studied.[32] It is fairly well established that the dynamics of very simple physical systems are quite complex if sufficient nonlinearity is present. Model equations of two-coupled Van der Pol oscillators are as follows:

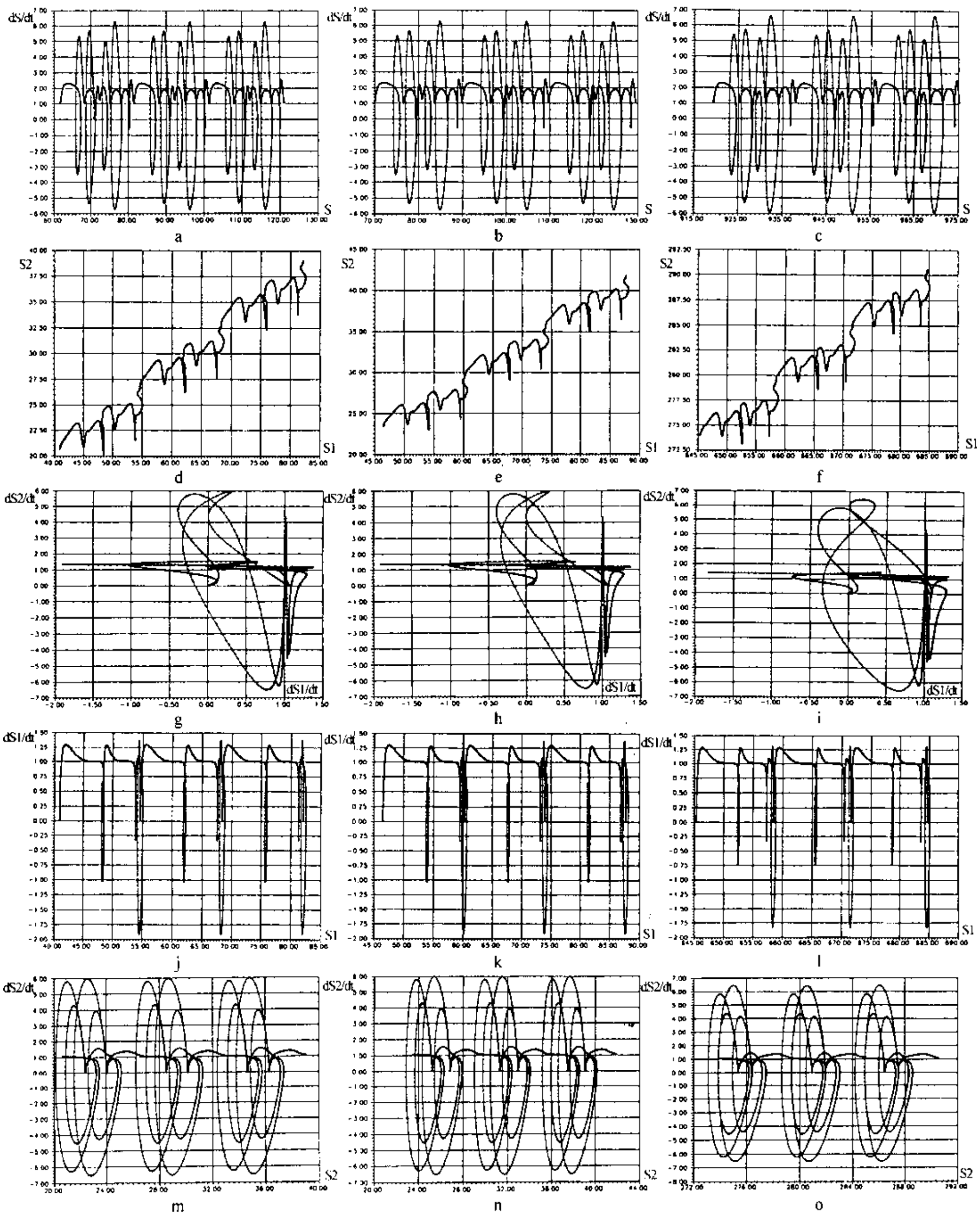

Fig. 7. Simulation results of temporal entropy-like behavior of two coupled Van der Pol oscillators: *a ~ c* - the phase portraits **(S~dS/dt)**; *d-f:* phase portraits (S_1-S_2); *g-l:* phase portraits (dS_1/dt - dS_2/dt); *j-l:* phase portraits (S_1-dS_1/dt); *m-o:* phase portraits **(S_2-dS_2/dt).**

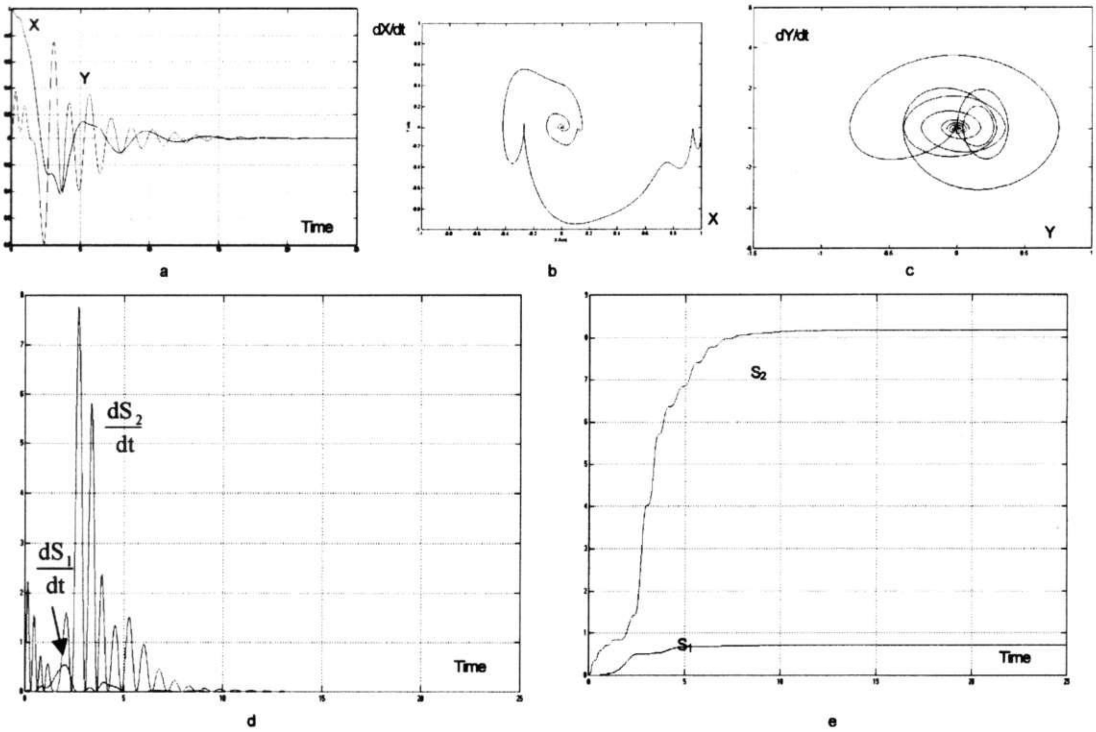

Fig. 8. Simulation results of temporal entropy-like behavior of nonlinear dynamic system (62): *a:* temporal behavior of mechanical motion (β_1=β_2=0.3; ω_1=1.5; ω_2=4; k=4; l=0.5) with initial states $x_0 = 1$; $y_0 = 0$; $\dot{x}_0 = \dot{y}_0 = 0$; *b:* phase portrait $(x, \dot{x})$; *c:* phase portrait $(y, \dot{y})$; *d:* temporal behavior of entropy production rates; *e:* temporal behavior of entropy production.

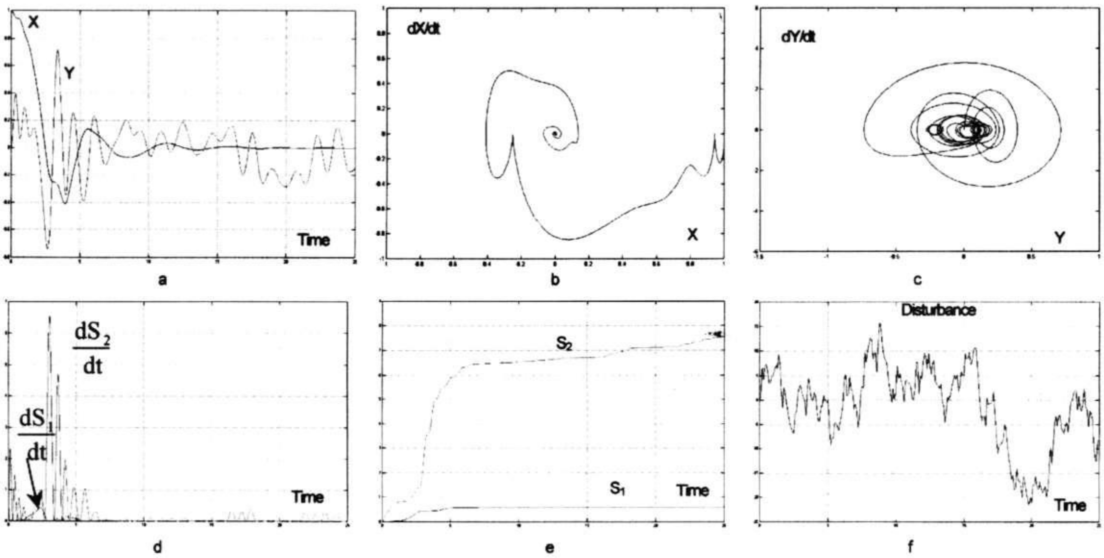

Fig. 9. Simulation results of temporal entropy-like behavior of nonlinear dynamic system (62): *a:* temporal behavior of mechanical motion (β_1=β_2=0.3; ω_1=1.5; ω_2=4; k=4; l=0.5) with initial states $x_0 = 1$; $y_0 = 0$; $\dot{x}_0 = \dot{y}_0 = 0$; M=5; *b:* phase portrait $(x, \dot{x})$; *c:* phase portrait $(y, \dot{y})$; *d:* temporal behavior of entropy production rates; *e:* temporal behavior of entropy production; f: random disturbance $U(t)$ with $\alpha = 0.5$; $\sigma_u^2 = 2$; $\omega = 1$.

$$\begin{cases} \ddot{x} + ([x + \beta y] - \xi_1)\dot{x} + (x + \beta y) = 0 \\ \ddot{y} + ([y + \alpha x] - \xi_2)\dot{x} + (y + \alpha x) = 0 \end{cases}, \quad \ldots\ldots \quad (60)$$

where α, β, ξ_1, and ξ_2 are parameters. The coupling considered is interpreted as a perturbation of oscillator amplitude through a signal proportional to the amplitude of the other.

In general, three cases (coupling in three regimes) consisting of values for ξ_1, ξ_2 = (0.1,1); (1,1); and (1,2) . For first case (0.1, 1), one oscillator is almost sinusoidal and other moderately nonlinear; two exactly moderately nonlinear oscillators; and one moderately nonlinear and the other quite so. For system (60), the characteristic polynomial is explicitly expressed[32] as

$$\lambda^4 - (\xi_1 + \xi_2)\lambda^3 + (2 + \xi_1\xi_2)\lambda^3 - (\xi_1 + \xi_2)\lambda + (1 - \alpha\beta) = 0, \quad \ldots\ldots\ldots \quad (61)$$

and depends only on three parameters ($\xi_1 + \xi_2$, $\xi_1\xi_2$, and $\alpha\beta$) and invariant under transformations ($\xi_1 \leftrightarrow \xi_2$, $\alpha \leftrightarrow \beta$). If $\alpha\beta = 1$, the manifold of equilibrium points is found to be a line in R^4, $\dot{x} = \dot{y} = 0$, $x = -\beta y$. For this special combination of coupling parameters, many infinitely new equilibrium points appear, and is a nonstandard feature of model equations studied. If $\alpha\beta < 0$, dynamics become richer in all cases studied. We concentrate mainly on the $\alpha\beta < 0$ region of the coupling parameter plane. The computation mode to clarify the attraction basins' structure for Eq.(60) concentrates mainly on two combinations of control parameters, called Case **A** and Case **B**.[32] Case **A** corresponds to ξ_1, $\xi_2 = 1.0$, $\alpha = [0.2, 0.7]$, $\beta = -1.75$ while Case **B** corresponds to $\xi_1 = 1.0$, $\xi_2 = 2.0$, $\alpha = [0.96, 1.0955]$, $\beta = -0.75$. Several attractors coexist in both cases, giving rise to a nontrivial structure of corresponding attraction basins. The main difference is that while, in Case **A**, symmetric asymmetric attractors coexist for some range of control parameter α, in Case **B**, only asymmetric attractors are found for $\alpha > 0.98$. Another difference is that, in Case **A**, all attractors are chaotic, while in Case **B**,[32] they begin with two asymmetric chaotic attractors (coming from the symmetry-breaking bifurcation), then two asymmetric limit cycles appear (for $\alpha \approx 1.0055$), and for higher values of both, chaotic limit cycle attractors suffer a series of bifurcations.

We discuss entropy-like behavior of a dynamic system in Case **A**. For α in intervals [0.2,0.48] and [0.52,0.7],[32] two asymmetric chaotic attractors are found, while for α in [0.483,0.51], another symmetric chaotic attractor is added. For α near symmetric attractor creation or destruction, the basin structure consists of zones where both basins are clearly separated and others where both basins and self-similarity occur at a smaller scale. The uncertainty exponent is significantly lower outside bifurcation ($\alpha \approx 0.5$) than for $\alpha = 0.5$, where a symmetric attractor is also found. **Figure 6** shows full dynamic behavior of two coupled Van der Pol oscillators. **Figure 7** compares entropy-like behavior between different strange attractors.

Consider simulation results of entropy-like behavior for dynamic systems as

$$\begin{cases} \ddot{x} + (2\beta_1)\dot{x} + \omega_1^2(1 - ky) = 0 \\ \ddot{y} + (2\beta_2)\dot{y} + \omega_2^2 y + \frac{\pi^2}{2l}(x\ddot{x} + \dot{x}^2) = \frac{1}{M}U(t) \end{cases}, \quad \cdots \quad (62)$$

where β_1, β_2, ω_1^2, ω_2^2, l, M, and k are parameters.

Figure 8 shows free motion and entropy-like behavior of Eq.(62) for $U(t) = 0$. Entropy production rates $\frac{d_1S}{dt} = 2\beta_1\dot{x}^2$; $\frac{d_2S}{dt} = 2\beta_2\dot{y}^2$ are equal to transport of the kinetic energy of both oscillators. Figure 8 shows the effect of kinetic energy (entropy) transport from x axis vibration to y axis vibration and vice versa.

Figure 9 shown stochastic entropy-like behavior of Eq. (62) when stochastic process $U(t)$ is a Gaussian random process with autocorrelation function $R_U(\tau) = \sigma_U^2 e^{-\alpha|\tau|}\cos\omega\tau$. In random excitation, the entropy production in dynamic system (62) decreases for both oscillators similar as in a one DOF nonlinear oscillator.[33] These results of entropy-like behavior simulation are used as a fitness function of GA in Parts 2 and 3 for optimal intelligent robust control.

5. Conclusions

Relations between the notion of the Lyapunov function (stability conditions), entropy production (thermodynamic behavior), and the physical realization of approximate mathematical models describing irreversible relaxation processes in closed nonlinear dissipative dynamic systems were studied. Thermodynamic criteria (positive entropy production rate) as a physical measure for realizing a mathematical model describing relaxation processes is introduced. These criteria indicate the necessity of putting extra (thermodynamic) limitations on parameters of differential equations and on symmetrics properties describing the evolution of nonlinear dynamic systems. A similar relation between Lyapunov functions and an entropy production rate in open dissipative dynamic systems with entropy structure exchange in Parts 2 and 3 were also studied.

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