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Principle of minimum entropy production in applied soft computing for advanced intelligent robotics and mechatronics

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Abstract A new approach to design of smart intelligent control systems for advanced robotics and mechatronics is developed. The principle of minimum entropy production in a control object motion and a control system as a fitness function for genetic algorithm is used. Simulation results of a smart robust control of non-linear systems described as coupled oscillators are presented.

Key words Applied soft computing, Intelligent control, the Principle of minimum entropy production

1 Introduction

Conventional computing basic tools for design of industrial intelligent control system includes fuzzy sets theory, fuzzy neural networks (FNN) and genetic algorithms (GA). Development of control systems of complex dynamic systems motion has brought to two researching ways: (1) the study of stable motion processes; and (2) an unstable motion processes of complex dynamic systems.

In the first case (of stable motion) the development and design of intelligent control algorithms can be described in the structure submitted in Fig. 1. The characteristic feature of the given structure is the consideration of the control object based on fuzzy system theory as a "black-box", and the study and optimization of an "input-output" linguistic relations using GA, FNN and fuzzy control (FC) to describe the changing law of PID-controller parameters with a minimum control error. At the small uncontrollable (unobservable) external excitations or small change of parameters (or structure) of control objects such approach guarantees the robust and stable control [2].

In a case of a global unstable dynamic control object such approach does not guarantee stable control in principle. For such unstable dynamic control objects new intelligent robust algorithms based on knowledge about a movement of essentially non-linear unstable dynamic systems are needed [4]. The structure of the intelligent robust control algorithms in general form for unstable dynamic control objects in Fig. 2 is shown. In Figs. 1 and 2 we are used next designations: GA – Genetic Algorithm; f – Fitness Function of GA; S – Entropy of System; S_c – Entropy of Controller; S_i – Entropy of Controlled Plant; ε – Error; u^* – Optimal Control Signal; m(t) – Disturbance; FC – Fuzzy Controller; FNN – Fuzzy Neural Network; FLCS – Fuzzy Logic Classifier System; SSCQ – Simulation System of Control Quality; K – Global Optimum Solution of Coefficient Gain Schedule (Teaching Signal); LPTR – Look-up Table of Fuzzy Rules; CGS – Coefficient Gain Schedule $k = (k_1, k_2, k_3)$.

This approach was firstly presented in [2, 3] as a new physical measure of control quality for complex non-linear controlled objects described as non-linear dissipative models. This physical measure of control quality is based on the physical law of minimum entropy production rate in intelligent control system and in the dynamic behavior of complex control object. This physical measure of control quality was used as a fitness function of GA in optimal control system design (see, Fig. 2, Box SSCQ).

The introduction of the new physical criteria (the minimum entropy production rate) guarantees the stability and robustness control of unstable objects [4]. This method differs from aforesaid design method (see, Fig. 1) in that a new intelligent global feedback in control system is introduced. The relation between the stability of control object (the Lyapunov function) and controllability (the entropy production rate) is used. The basic feature of the given method is the necessity of approximate model investigation for control object and the calculation of the entropy production rate through the parameters of the developed model. The method of the accuracy evaluation for a model approximation using the entropy approach in [3] is presented. The integration of joint systems of equations (the equations of mechanical model motion and the equations of entropy production rate) enable to use the result as the fitness function in GA.

The general approach to design method of robust intelligent control for complex non-linear unstable control

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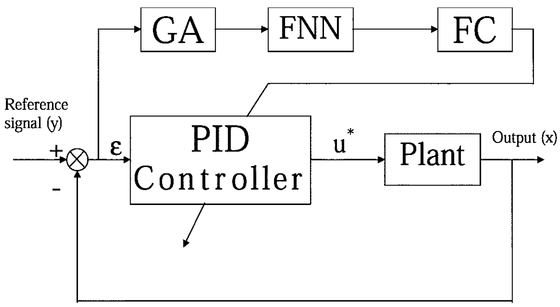

Fig. 1. Structure of A1 control system (Designation see in text)

objects based on soft computing (using the principle of minimum entropy production rate) is described. The results of entropy-like dynamic behavior modeling of the typical Benchmarks of dynamic control systems are presented.

2
Entropy production of the relaxation irreversible processes in closed dynamic systems

We will use the phenomenological thermodynamic approach developed in [2] for the analysis of any class of the dynamic control systems described by nonlinear dissipative differential equations. Let us investigate the relations between the notion of the Lyapunov function, entropy production rate and the physical realization of approximate mathematical models describing irreversible processes in closed nonlinear dynamic systems.

From the thermodynamic standpoint consider two cases of behavior of dynamic systems [2]: irreversible processes of generalized forces, and irreversible processes of generalized coordinates. Begin from the first case.

2.1
Entropy production in irreversible processes of generalized forces: generalized case

Let us consider the motion of the dynamic system as a relaxation process described by the generalized equation [2]

$$X_a = f_a(y, \dot{y}, T), \quad a = 1, 2, \ldots, m \ , \tag{1}$$

where X_a are generalized forces, $y = (y_1, y_2, \ldots, y_n)'$ is a vector of generalized coordinates, $\dot{y} = (\dot{y}_1, \ldots, \dot{y}_n)'$ is a vector of generalized velocities, T is a temperature.

In Eq. (1) we suppose that a vector-function $f_a(\ldots)$ is an analytical function and admits the expansion in an absolutely convergent power series. In such an event

$$f_a(y, \dot{y}, T) = \underbrace{\alpha_0^a + \sum_i \alpha_i^a y_i + \sum_{i,k} \alpha_{ik}^a y_i y_k + \sum_{i,k,p} \alpha_{ikp}^a y_i y_k y_p + \cdots +}_{\text{non-dissipative}} \underbrace{+ \sum_i \beta_i^a \dot{y}_i + \sum_{i,k} \beta_{ik}^a \dot{y}_i y_k + \cdots + \sum_{i,k,p} \gamma_{ikp}^a \dot{y}_i \dot{y}_k y_p + \cdots}_{\text{dissipative}} \tag{2}$$

On this assumption the expansion in Eq. (2) is an absolutely convergent power series. Introduce some designations:

$$F_a(y, T) = \alpha_0^a + \sum_i \alpha_i^a y_i + \sum_{i,k} \alpha_{ik}^a y_i y_k + \cdots \ , \tag{3}$$

$$\Psi_a(y, \dot{y}, T) = \sum_i \beta_i^a \dot{y}_i + \sum_{i,k} \beta_{ik}^a \dot{y}_i y_k + \cdots \tag{3a}$$

Obviously that $\Psi_a(y, 0, T) = 0$. We can transform Eq. (1) in accordance with Eqs. (3) and (3a) to

$$X_a = F_a(y, T) + \Psi_a(y, \dot{y}, T) \ . \tag{4}$$

If the system (1) has been in a state of equilibrium, then

$$X_a = F_a(y, T) \ . \tag{5}$$

If the relation of Eq. (5) does not hold with $X_a = \text{const}$ or if $X_a = X_a(t)$, i.e. dependent from time t, then we must have much more generalized relations from Eq. (4). One can see from Eq. (4) that a generalized thermodynamic force X_a is the additive function consisting from two parts: a "reversible" part $X_a^r = F_a(y, T)$ and an "irreversible" part $X_a^{ir} = \Psi_a(y, \dot{y}, T)$. Thus according to the phenomenological thermodynamics, we define the entropy production rate as the following:

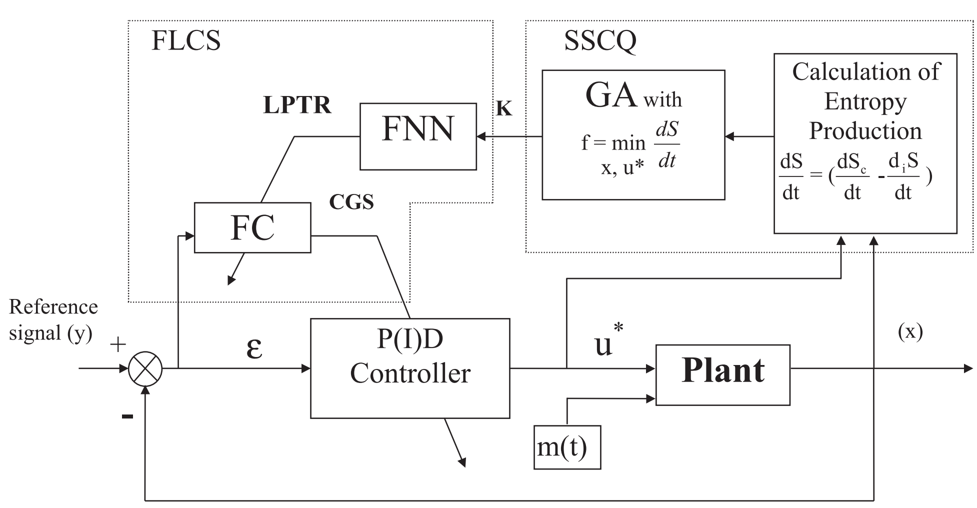

Fig. 2. Structure of self-organization A1 control system with the physical measure of control quality (Designation see in text)

$$\frac{\mathrm{d}_i S}{\mathrm{d}t} = \sigma = \frac{1}{T}\sum_a \Psi_a(y_1, \ldots, y_n, \dot{y}_1, \ldots, \dot{y}_n, T)\dot{y}_a > 0 \quad . \tag{6}$$

Considering that $T > 0$, from Eq. (6) we get the following thermodynamic conditions for the physical realization of a nonlinear dynamic system (4)

$$\sum_a \Psi_a(y_1, \ldots, y_n, \dot{y}_1, \ldots, \dot{y}_n, T)\dot{y}_a > 0, \quad \forall\, y_k \,\&\, \dot{y}_i \quad . \tag{7}$$

Specifically, if the function $\Psi_a(\ldots)$ is a linear function, then the thermodynamic criteria (7) offers to the requirement of the positive-definite quadratic form.

It must be pointed out that the equilibrium in Eq. (4) must be able to admit the following written form

$$X_a = F_a(y, T) = \frac{\partial F(y, T)}{\partial y_a} \tag{8}$$

that is X_a in (8) must be a potential force. In Eq. (8) the function $F(y, T)$ is a free energy of the system (1).

Notice that only with all of above listed thermodynamic restrictions the initial system (1) can be physically realized.

Let us consider without the restriction of a generality that $X_a = 0$. In this case we deal with the relaxation of generalized coordinates y_i and the relaxation process is described by the following equations:

$$F_a(y, T) + \Psi_a(y, \dot{y}, T) = 0, \quad T = \text{const} \quad , \tag{9}$$

i.e., we deal with the isothermal relaxation. The relaxation is an irreversible process and must fulfill the following equality

$$\frac{\mathrm{d}_i S}{\mathrm{d}t} = \frac{1}{T}\sum_a \Psi_a(y, \dot{y}, T)\dot{y}_a > 0 \quad , \tag{10}$$

i.e., during the relaxation we get an increase of entropy with the decrease of entropy production.

2.2 Interrelation between entropy production and Lyapunov function of irreversible processes in closed systems

Suppose that in the considered domain of variable change $(y, \dot{y})$ we have the inequality: $F > 0$. Identify in this domain the free energy F with a Lyapunov function V, i.e., $F \equiv V$.

Theorem With the above assumption the entropy production $\sigma = \mathrm{d}_i S/\mathrm{d}t$ in a relaxation process of the system (1) and Lyapunov function V have the following correlation

$$\sigma = -\frac{1}{T}\frac{\mathrm{d}V}{\mathrm{d}t} \quad . \tag{11}$$

Proof. According to Eq. (8) the correlation in Eq. (9) can be rewritten as

$$\frac{\partial F(y, T)}{\partial y_a} + \Psi_a(y, \dot{y}, T) = 0, \quad T = \text{const} \quad . \tag{12}$$

After multiplying both parts in Eq. (12) by $\dot{y}_a$ and calculation on index "a" we obtain the following equation:

$$\sum_a \left[\frac{\partial F(y, T)}{\partial y_a}\dot{y}_a + \Psi_a(y, \dot{y}, T)\dot{y}_a\right] = 0, \quad T = \text{const} \quad . \tag{13}$$

We can write

$$\frac{\mathrm{d}V}{\mathrm{d}t} = \frac{\partial V}{\partial y_a}\dot{y}_a = \frac{\partial F}{\partial y_a}\dot{y}_a \quad , \tag{14}$$

$$\sum_a \Psi_a(y, \dot{y}, T)\dot{y}_a = T\frac{\mathrm{d}_i S}{\mathrm{d}t} = T\sigma \quad .$$

Therefore, Eq. (14) can be written in the following form

$$\sigma = -\frac{1}{T}\frac{\mathrm{d}V}{\mathrm{d}t} \quad , \tag{15}$$

i.e., Eq. (11) Q.E.D.

The Eq. (11) is one of generalization in the stability theory of a relaxation process.

This relation is a result of an irreversible process in relaxation of thermodynamic forces.

2.3 Entropy production in irreversible processes of generalized coordinates

Let us consider a dynamic system with generalized coordinates x_k described as the sum of a "reversible" x_k^r and an "irreversible" x_k^{ir} parts,

$$x_k = x_k^r + x_k^{ir} \quad . \tag{16}$$

Let that for isothermal process ($T = \text{const}$) indicial equations can be described as the following: for reversible equilibrium parts of generalized coordinates as

$$x_k^r = \sum_j A_{kj}X_j \tag{17}$$

and for irreversible non-equilibrium parts of generalized coordinates as

$$\frac{\mathrm{d}x_k^{\mathrm{ir}}}{\mathrm{d}t} = f_k(X_1, X_2, \ldots, X_n, T) \quad . \tag{18}$$

We suggest that for coefficients A_{kj} in Eq. (17) and a function f_k in Eq. (18) the following relations are true

$$\sum_{k,j} A_{kj}X_kX_j > 0, \quad A_{kj} = A_{jk}, \quad f_k(0, 0, \ldots, 0, T) = 0 \tag{19}$$

After differentiation of Eq. (16) in time and according to Eqs. (18) and (19) we can obtain the following kinetic equations for the isothermal process

$$\frac{\mathrm{d}x_k}{\mathrm{d}t} = \sum_j A_{kj}\frac{\mathrm{d}X_j}{\mathrm{d}t} + f_k(X_1, X_2, \ldots, X_n, T) \quad . \tag{20}$$

According to the definition of generalized thermodynamic forces and corresponding to them generalized coordinates the entropy production rate is equal to

$$\sigma = \frac{d_i S}{dt} = \frac{1}{T}\sum_j X_j \frac{dx_j^{ir}}{dt} = \frac{1}{T}\sum_k X_k f_k(X_1, \ldots, X_n, T) > 0 \ . \quad (21)$$

According to the second law of thermodynamic an entropy production rate σ connected with an irreversible process must be positive. Equation (21) is true for $T > 0$. Thus the kinetic equations (20) are true if for the functions $f_k(\ldots)$ the relations (19) are true. It is a requirement of the thermodynamic criteria of physical realization for a mathematical model to be described as an irreversible process, in control objects.

3 Relaxation irreversible processes with entropy exchange

3.1 Entropy production in kinetic equations of relaxation processes

Consider the case when an entropy production is a scalar parameter in kinetic equations of relaxation processes. The physical meaning of this case was discussed in [2]. For an adiabatic isolated system generalized thermodynamic forces can be described as

$$X = F(x, \dot{x}, S) \ , \quad (22)$$

where S is an instantaneous value of an entropy. For analytical function $F(x, \dot{x}, S)$ as for Eq. (4) we can write

$$X = F^r(x, S) + F^{ir}(x, \dot{x}, S) \quad (23)$$

where a “reversible” part $X^r = F^r(x, S)$ and an “irreversible” part $X^{ir} = F^{ir}(x, \dot{x}, S)$, and, what’s more, $F^{ir}(x, 0, S) = 0$.

The general system of equations describing compound parts “mechanical + thermodynamic” behavior in dynamic systems carried out as following:

$$X = F^r(x, S) + F^{ir}(x, \dot{x}, S), \ \frac{d_i S}{dt} = \frac{1}{T} F^{ir}(x, \dot{x}, S)\dot{x},$$
$$T = \varphi(S) \ \text{(the terminal thermodynamic relation)},$$
$$\dot{x}F^{ir}(x, \dot{x}, S) > 0 \ \text{(the physical restriction)} \ . \quad (24)$$

The solution of Eqs. (24) includes a history of entropy S-evolution and describes an evolution of system (24) taken into account a self-degeneracy with increase of the entropy motion.

Example Let a dynamic system has kinetic energy T_k and potential energy U, and its dynamic behavior is described by Lagrangian equations as

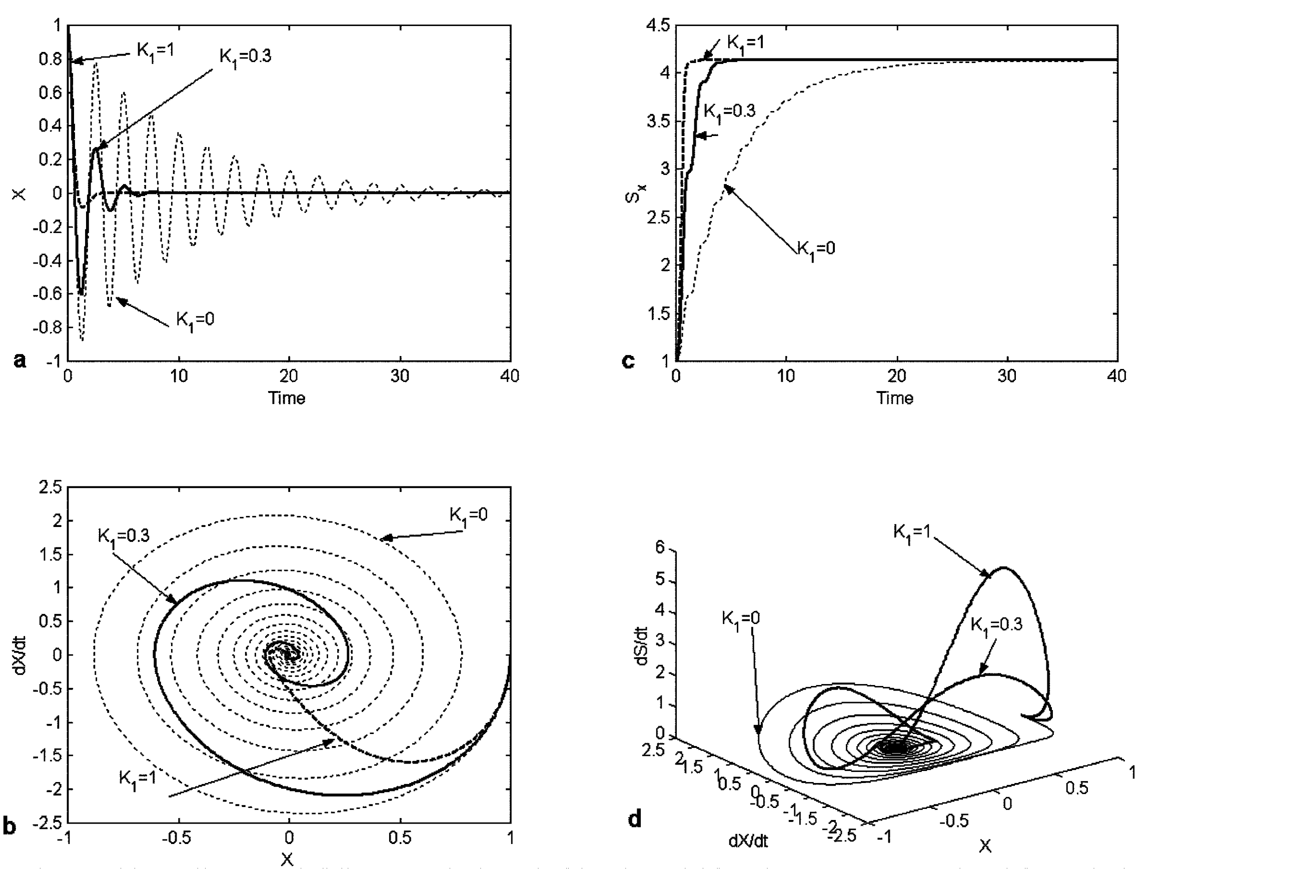

Fig. 3a–d. Dynamic (**a**, **b**) and thermodynamic (**c**, **d**) behavior of nonlinear system described by (26) with different dissipative parameter K_1

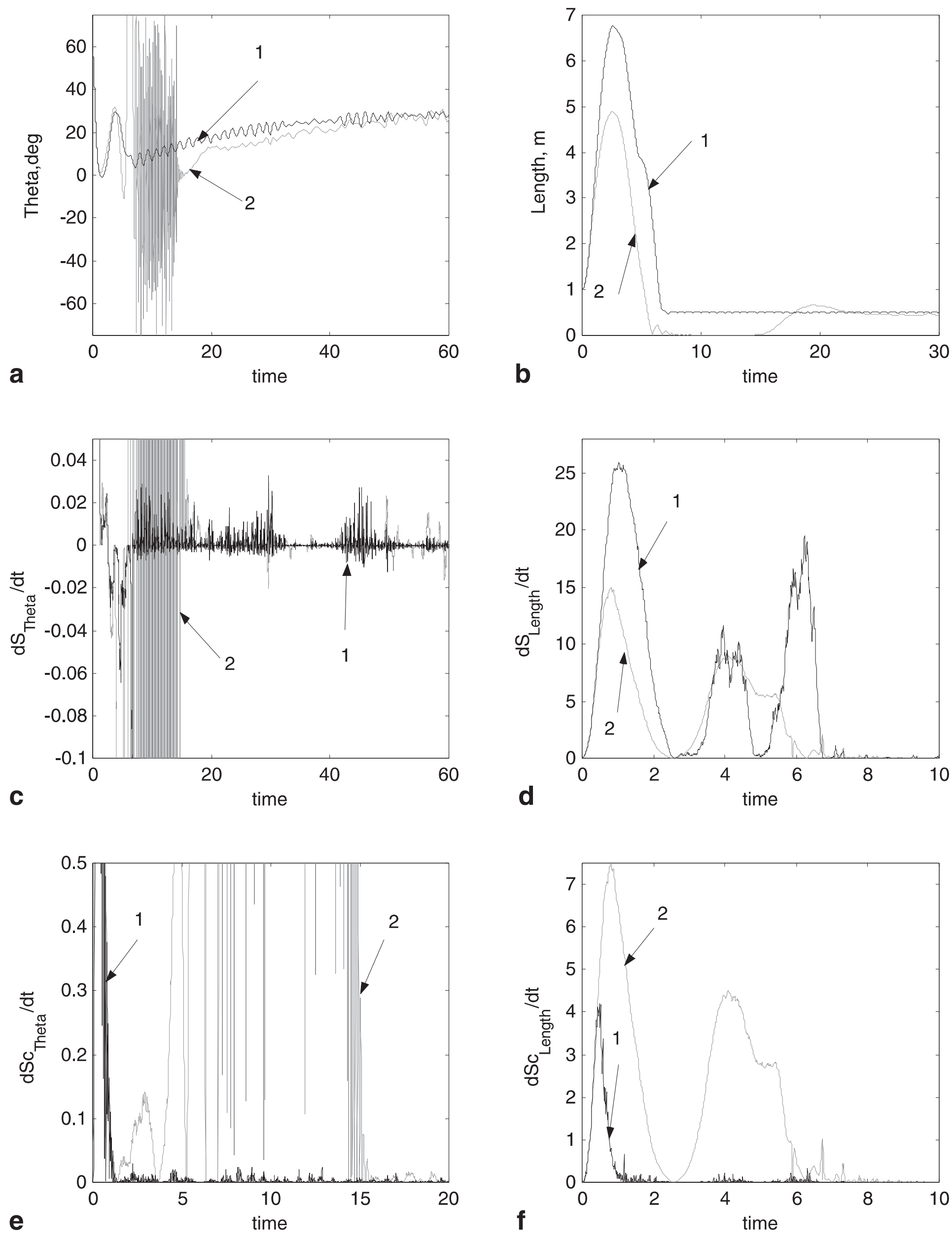

Fig. 4a–f. Simulation results of dynamic and thermodynamic behavior of swing system

$$\frac{\mathrm{d}}{\mathrm{d}t}\frac{\partial T_k}{\partial \dot{q}_i} - \frac{\partial T_k}{\partial q_i} = \frac{\partial U}{\partial q_i} + Q_i(q_i, \dot{q}_i, S),$$
$$\frac{\mathrm{d}_i S}{\mathrm{d}t} = \sigma = \frac{1}{T}\sum_i \dot{q}_i Q_i(q_i, \dot{q}_i, S) \ , \qquad (25)$$

$S = F(q, T)$ (thermodynamic equation of coupling), $\sum_i \dot{q}_i Q_i(q_i, \dot{q}_i, S) > 0$ (identically) Equations (25) describe the case when a heat exchange interaction between the dynamic system and external world is absent, and $Q_i(q_i, \dot{q}_i, S)$ are non-conservative forces including as a particular case dissipative forces. In this case an entropy production completely is caused by a self-degeneracy of a mechanical system motion.

For the case when dissipative forces are dependent linearly from entropy S as $Q(q, \dot{q}, S) = (k + k_1 S)\dot{q}$ we can write the equation of motion for dynamic system with one degree of freedom from Eqs. (25) as following:

$$\ddot{q} + (k + k_1 S)\dot{q} + k_0 q = A \sin k_0 t \ ,$$
$$\frac{\mathrm{d}_i S}{\mathrm{d}t} = \frac{1}{T}(k + k_1 S)\dot{q}^2, \quad S_0 = c_q \ln T + \alpha_k q_0 \qquad (26)$$

From Eqs. (26) follows that the dynamic system (26) is described by the time-variant non-linear structure. Figure 3 shows the simulation results of an entropy accumulation in the dynamic system motion according to Eqs. (26). They show the high sensitivity of a thermodynamic behavior to a change of the entropy parameter in a mechanical system motion.

Using of the entropy as the scalar parameter in a mechanical system motion was described firstly in [2] and was repeated in [1].

3.2 Dynamic stability of systems with control entropy exchange

Consider a dynamic system that has an entropy exchange with a heat reservoir (a control system). This is the generalized case of Eqs. (24) and (25) when the entropy of the dynamic system is $S = S^p - S^{ex}$, where S^p is the entropy produced by the dynamic system, and S^{ex} is the exchange entropy with the heat reservoir. In this case Eqs. (24) can be written as

$$\begin{aligned} &X = F^r(x, S) + F^{ir}(x, \dot{x}, S), \\ &S = S^p - S^{ex}, \quad \frac{dS^p}{dt} = \frac{d_i S}{dt} = \frac{1}{T} F^{ir}(x, \dot{x}, S)\dot{x} \ , \\ &\frac{dS^{ex}}{dt} = \psi(x, \dot{x}, S) \ , \\ &\varphi(S, T, x) = 0, \quad \dot{x}_k = f_k(x, S, t) \ . \end{aligned} \tag{28}$$

Consider a new Lyapunov function

$$V = \frac{1}{2}\left(\sum_{k=1}^{n} x_k^2 + S^2\right) \tag{29}$$

After the differentiation of Eq. (29) and the simple algebraic transformation we have

$$\frac{dV}{dt} = \underbrace{\sum_{k=1}^{n} x_k f_k(x, S, t)}_{\text{mechanical motion}} + \underbrace{(S^p - S^{ex})\left(\frac{dS^p}{dt} - \frac{dS^{ex}}{dt}\right)}_{\text{thermodynamical behavior}} \tag{30}$$

Equation (30) describes the generalized interrelation between mechanical behavior (Lyapunov stability) and entropy production in open dynamic systems.

4 Simulation results: intelligent control of swing system (pendulum with variable length)

Consider the motion of the swing system under control described by the following equations:

$$\begin{cases} \ddot{\theta} + 2\frac{\dot{l}}{l}\dot{\theta} + \frac{g}{l}\sin\theta = k_1 \cdot e_\theta + k_2 \cdot \dot{e}_\theta + k_3 \cdot \int e_\theta \, dt + \xi(t) \\ \ddot{l} + 2k\dot{l} - l\dot{\theta}^2 - g\cos\theta = \frac{1}{m}\left(k_4 \cdot e_l + k_5 \cdot \dot{e}_l + k_6 \cdot \int e_l \, dt + \xi(t)\right) \end{cases} \tag{31}$$

Here $\xi(t)$ is the given stochastic excitation (white noise). Equations of entropy production are the following:

$$\frac{dS_\theta}{dt} = 2\frac{\dot{l}}{l}\dot{\theta} \cdot \dot{\theta}; \quad \frac{dS_l}{dt} = 2k\dot{l} \cdot \dot{l} \tag{32}$$

The system (31) is globally unstable system (in Lyapunov sense). Simulation results of dynamic motion under the control are presented in Fig. 4a, b. Two types of control approaches (with Soft Computing as GA with fitness function as minimum of entropy production rate for SGC of conventional PID controller, curve – 1; conventional PID control with fixed gain coefficients, curve – 2) are compared. The thermodynamic behavior is shown in Fig. 4c, d, e, f where Fig. 4c, d are the entropy production rates of state variables θ and l respectively. In Fig. 4e, f the entropy production rates of two PID controllers are shown. Figure 3a and b shown that intelligent control more robust and effective for unstable control object in presence of random excitations.

5 Conclusions

The interrelations between the notion of the Lyapunov function (stability conditions), entropy production (thermodynamic behavior) and the physical realization of approximate mathematical models describing an irreversible relaxation processes in closed and open nonlinear dissipative dynamic systems are investigated. The thermodynamic criteria (as the minimum of entropy production rate) as a physical measure for a fitness function of GA is introduced. Design of robust intelligent control of complex non-linear dynamic systems based on soft computing is demonstrated. The effectiveness of this approach by the simulation of Benchmark is shown.

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