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ARTIFICIAL INTELLIGENCE

Quantum Fuzzy Inference for Knowledge Base Design in Robust Intelligent Controllers

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Abstract—The analysis of simulation results obtained using soft computing technologies has allowed one to establish the following fact important for developing technologies for designing robust intelligent control systems. Designed (in the general form for random conditions) robust fuzzy controllers for dynamic control objects based on the knowledge base optimizer (stage 1 of the technology) with the use of soft computing can operate efficiently only for fixed (or weakly varying) descriptions of the external environment. This is caused by possible loss of the robustness property under a sharp change of the functioning conditions of control objects: the internal structure of control objects, control actions (reference signal), the presence of a time delay in the measurement and control channels, under variation of conditions of functioning in the external environment, and the introduction of other weakly formalized factors in the control strategy. In this paper, a description of the strategy of designing robust structures of an intelligent control system based on the technologies of quantum and soft computing is given. The developed strategy allows one to improve the robustness level of fuzzy controllers under the specified unpredicted or weakly formalized factors for the sake of forming and using new types of processes of self-organization of the robust knowledge base with the help of the methodology of quantum computing. Necessary facts from quantum computing theory, quantum algorithms, and quantum information are presented. A particular solution of a given problem is obtained by introducing a generalization of strategies in models of fuzzy inference on a finite set of fuzzy controllers designed in advance in the form of new *quantum fuzzy inference*. The fundamental structure of quantum fuzzy inference and its software toolkit in the processes of designing the knowledge base of robust fuzzy controllers in on-line, as well as a system for simulating robust structures of fuzzy controllers, are described. The efficiency of applying quantum fuzzy inference is illustrated by a particular example of simulation of robust control processes by an essentially nonlinear dynamic control object with randomly varying structure.

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INTRODUCTION

In complex and essentially nonlinear dynamic models of control objects (CO) with weakly formalized structure and random parameters, it is quite difficult to determine an optimal structure of an automatic control system (ACS), in which, e.g., a conventional proportional–integral–differentiating (PID) controller is employed at the lower (executive) level. Especially, this difficulty reveals itself in problems of designing the structures of automatic control systems of this type in the presence of random noise different in its nature and under uncertainty information about the control goals.

In the design methodology of intelligent control systems (ICS) based on soft computing technology, the structure of a fuzzy controller (FC) is considered as one of the variants of designing conventional automatic control systems. Beginning from the moment of origination (1974), the models of fuzzy controllers have demonstrated improved ability of controlling dynamic control objects that have a weakly formalized structure or function in the conditions of uncertainty of the information source. As the experience and simulation results have shown that, in the listed control situations frequently, a conventional (based on the principles of a global negative feedback) PID controller cannot manage the control problem posed. The use of the soft computing technology (based on genetic algorithms (GA) and fuzzy neural networks (FNN)) has extended the field of efficient application of fuzzy controllers by using new functions in the form of learning and adaptation. However, it is very difficult to design a globally appropriate and robust structure of the intelligent control system. This limitation is especially typical of unpredicted control situations, when the control objects operates in sharply changing conditions (a failure of sensors or noise in the sensor system, the presence of a time delay in control signals or measurement, a sharp change in the structure of the control objects or its parameters, etc.).

In a number of practical cases, conditions of this type can be predicted, but it is difficult to realize a robust control in unpredicted situations based on the designed (for a fixed situation) knowledge base (KB) of a fuzzy controller (even of the whole set of predicted random situations). It seems to us that one of the existing solutions is to form a finite number of knowledge bases of the fuzzy controller for the set of fixed control situations.

The question arises about *how to determine which of knowledge bases can be used at a particular time instant.* In this case, the choice of a generalized strategy to provide the opportunity to switch the flow of control signals input from different knowledge bases of the fuzzy controller and to adapt their output signal (if necessary) to the current conditions of functioning of the knowledge bases of a control object is especially important. A simple variant of solving this problem is to use the method of weighted coefficients and aggregate the output signals from each independent fuzzy controller. Regrettably, this method has limited capabilities (as simulation results have shown) since, frequently, the distribution of weighting factors has to be determined in on-line dynamics (see in what follows and [1, 2]), and the search procedure has combinatorial nature.

In this paper it is shown that a solution of the problems of this type can be found based on introducing the self-organization principle in the course of designing knowledge bases of a fuzzy controller, which is implemented and has a software toolkit using the developed model of quantum fuzzy inference with application of the methodology of quantum soft computing and systems engineering (System of Systems Engineering) using the synergetic self-organization principle [3–5].

In particular, the implementation of the process of self-organization of robust knowledge bases in this approach is performed using generalization of strategies of fuzzy logical inference in the form of quantum fuzzy inference. The structure of quantum fuzzy inference and a system for simulating robust knowledge bases for fuzzy controller, which illustrate the efficiency of application of quantum fuzzy inference, are described. The model of the developed quantum fuzzy inference is regarded as a new type of the search quantum algorithm on the generalized space of knowledge bases of a fuzzy controller, and a generalized robust control signal is designed as the output result.

The model of quantum fuzzy inference proposed in this paper harness particular individual knowledge bases of a fuzzy controller, each of which is obtained with the help of an knowledge base optimizer (KBO) for the corresponding conditions of functioning of a control object and fixed control situations in a random external environment. The process of designing particular individual knowledge bases of a fuzzy controller with the help of the knowledge base optimizer for given control situations is performed in accordance with the design technology and is considered in detail in [1, 6] (see stage 2 in Fig. 3 in [7]).

In particular in [7], based on a comparison of simulation results obtained in [1, 6], it was shown that for sufficiently wide range of variation of parameters characterizing a *given* control situation, the knowledge base optimizer yields an essential gain (compared with other software tools) in order to achieve the required robustness level of knowledge bases. Other industrial tools for forming knowledge bases, such as the FNN ANFIS (built-in module in the Matlab simulation system) or AFM (an ST Microelectronics development [6]), etc. have an increased sensitivity to the variation of parameters (characterizing a given control situation) compared with the KBO tools and result in the loss of control robustness as was strictly shown in [7]. As a result, in fixed control situations, fuzzy controllers with knowledge bases (designed with the help of the Knowledge base optimizer) have improved robustness, and the corresponding control laws contain less *redundant information* and thus are used as an input signal for quantum fuzzy inference (in accordance with the developed technology of designing robust knowledge bases (Fig. 3 in [7]). It is worth noting that the presence of redundant information in control laws is physical reality, which takes place because of the use in the processes of optimization of knowledge bases of a random choice in the form of genetic algorithms, as well as it follows from the laws of information theory on the necessity of presence of redundancy in unreliable data transmission channels with noise. This is the inevitable cost of the possibility to obtain a solution of the problem of optimal control of an essentially nonlinear control plant in the conditions of uncertainty in the source information and the multi-criteria nature of the optimization conditions.

In the case of an unpredicted control situation, the additional information redundancy in the control laws of the fuzzy controllers arises as the total result of the inadequate reaction of the CO (control plant) (in the form of a new control error) and of the logically incorrect interpretation of the initialization of the corresponding production rules in the Knowledge bases used by the fuzzy controllers (trained only on given control situations).

The model of quantum fuzzy inference is a new type of the quantum search algorithm on the generalized space of structured data and, based on the methods of quantum computing theory [8–10], it allows one to solve efficiently control problems that could not be solved earlier at the classical level. The developed approach is first applied in the theory and practice of intelligent control systems. Therefore the present paper is a generalization and development of the results obtained in [1, 2, 6, 7].

Remark 1. From the point of view of the complexity theory of computations and the quantum algorithm [8–10], the developed quantum algorithm (QA) is referred to the class of polynomial with bounded error, the BPP class (bounded-error probabilistic polynomial time), and its quantum generalization is referred to the BQP class. Therefore, it is efficient by the definition. This means that, structurally, the quantum algorithm in quantum fuzzy inference has a polynomial complexity; i.e., the randomized algorithm has polynomial, rather than exponential (as in classical search algorithms) dependence on the input signals, and a bounded probability (at the level 3/4) of the measurement accuracy of the result of computations is sufficient for (making) an efficient decision making.

In the proposed model of the quantum algorithm for quantum fuzzy inference the following actions are realized [3–5]: (1) the results of fuzzy inference are processed for each independent FC fuzzy controller; (2) based on the methods of quantum information theory, valuable quantum information hidden in independent (individual) knowledge bases is extracted; and (3) in on-line, the generalized output robust control signal is designed in all sets of knowledge bases of the fuzzy controller.

In this case, the output signal of quantum fuzzy inference in on-line is an optimal signal of control of the variation of the gains of the PID controller, which involves the necessary (best) qualitative characteristics of the output control signals of each of the fuzzy controllers, thus implementing the self-organization principle. Therefore, the domain of efficient functioning of the structure of the intelligent control system can be essentially extended by including *robustness*, which is a very important characteristic of control quality. The robustness of the control signal is the ground for maintaining the reliability and accuracy of control under uncertainty conditions of information or a weakly formalized description of functioning conditions and/or control goals [7].

In this paper, we describe the fundamental structure of quantum fuzzy inference and its software toolkit in the processes of designing knowledge bases of robust self-organized fuzzy controllers in on-line. In particular, the functional organization of the system for simulating robust knowledge bases for fuzzy controllers that allows one to improve the efficiency of applying quantum fuzzy inference is presented. Necessary facts from quantum computing theory and quantum information theory used for development and substantiation of the structure of the quantum algorithm in quantum fuzzy inference are also presented.

1. STATEMENT OF THE PROBLEM

One of the main problems of the modern technology of designing fuzzy controllers is the design and application of robust knowledge bases in the structures of intelligent control system [1] in order to increase their ability to be self-learned (trained), self-organized, and self-developed. In connection with this fact, we pose in this paper the following problem: develop a model of quantum fuzzy inference for designing robust knowledge bases in intelligent controllers that ensure the achievement of a guaranteed control in unpredicted (abnormal) control situations. In the end, the solution of this problem results in increasing the robust level of the structure of the intelligent control system.

The problem of designing the structure and knowledge bases of the most intelligent fuzzy controllers for a given control situation were considered in previous publications of the authors (see a detailed description in [1, 6, 7], and are used in this paper as source data. In particular, in [7, Figs. 1 and 2], the interrelation between the measures of quality control and the types of intelligent computing tools was presented in detail. The relations between stability, controllability, and robustness were also investigated based on the thermodynamic approach [7, Fig. 1, level 1, and Fig. 3]. The corresponding quantitative measures and regularities were included in the software toolkit (support) of the knowledge base optimizer.

In the modern control theory, various aspects of learning and adaptation processes of fuzzy controllers were investigated. Many of the learning schemes were based on the error back propagation algorithm and their modifications [1]. The adaptation processes are based on iterative models of random algorithms. These ideas work well in designing control processes in the conditions of absence of weakly formalized noise of the external environment or unknown noise in the sensor system, etc. In more complex unpredicted control situations, the learning and adaptation methods using error back propagation methods or iterative random algorithms did not guaranteed the achievement of the required robustness and accuracy level of control processes. The efficient solution to this problem with the help of the knowledge base optimizer for particular control situations was developed in [1, 2, 7].

It was shown in [3] that to achieve self-organization [7, Fig. 1, level 3] in the structure of intelligent control systems, it is necessary to use quantum fuzzy inference. Figure 1 presents the general block diagram of quantum fuzzy inference. The principles of operation of quantum fuzzy inference and its particular blocks are considered in detail in Section 5 below.

The table presents the structure of the intelligent control system including the model of quantum fuzzy inference and describes its advantages and disadvantages. The model of quantum fuzzy inference is based on the physical laws of quantum computing theory [8–10], namely, unitary, and invertible quantum operators are used. In the general form, the quantum algorithm consists of three basic unitary operations: superposition, quantum correlation (quantum oracle and entangled operators) and interference. The fourth operator, the operator of measurement of the results of quantum computing is irreversible (classical).

Quantum computing based on the listed types of operators is referred to a new type of computational intelligence [10]. In what follows (to provide a more complete understanding of the principles of operation of the quantum algorithm in quantum fuzzy inference), we give a brief description of the listed quantum operators, their interrelations and properties. The necessary facts from quantum information theory and the theory of quantum correlation processes are presented. This information allows one to understand more completely and deeply the solution of the following problem that is difficult and fundamentally important for the theory and control systems: *determine the role and influence of quantum effects on the improvement of the robust level of the designed intelligent control processes* for the sake of extracting additional quantum information hidden (and accessible only partially) in the correlation classical states of control laws and designed only based on classical methods of soft computing. Additional information and a detailed presentation of these problems with a mathematical proof of the required propositions can be found in [10].

2. QUANTUM COMPUTING: EXAMPLES AND PROPERTIES OF THE BASIC OPERATORS

As an example, we consider the conventional mathematical formalism of describing models of the basic quantum operators from the point of view of the second quantum problem of describing the quantum algorithm (see Subsection 5. 2). This formalism can be expressed in the language of quantum states or transformations, but we are also interested in the possibility of an adequate description of quantum states and effects in the language of logical inference: the application of the conventional formalism, its power, and expressive power as a *quantum system of fuzzy logical inference* [10, 11].

Example 1. Quantum bit as a quantum state. A conventional bit may be in one of the two states, 0 or 1. Thus, its physical state can be represented as $b = a_1 0 + a_2 1$, which has one of the following forms: either $a_1 = 1$ and $a_2 = 0$, then $b = 0$, or $a_1 = 0$ and $a_2 = 1$, then $b = 1$. On the contrary, the state of a quantum bit $|\psi\rangle$ is given by a vector in a two-dimensional complex space. Here, the vector has two components, and its projections on the bases of the vector space are complex numbers. The quantum bit is represented (in the Dirac notation in the form of a ket vector) as $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$ or in the vector notation $|\psi\rangle = \begin{bmatrix}\alpha\\ \beta\end{bmatrix}$, $\langle\psi| = \begin{bmatrix}\alpha & \beta\end{bmatrix}^T$ (bra vector). If $|\psi\rangle = |0\rangle$, then $|0\rangle = \begin{bmatrix}1\\ 0\end{bmatrix}$.. The amplitudes α and β are complex numbers such that the condition $\alpha\alpha^* + \beta\beta^* = 1$ holds, where ì*î is the operation of complex conjugation; and $|0\rangle$ and $|1\rangle$ form a pair orthonormal basis vectors called a *state of the computational basis.* If α and β take zero values, then ψ defines a classical, pure state. Otherwise, it is said that ψ is in the state of superposition of two *classical* basis states.

Geometrically, the quantum bit is in a continuous state between $|0\rangle$ and $|1\rangle$ until its state is measured. The notion of amplitude of the probabilities of the quantum state is a combination of the concepts of state and phase. In the case when the system consists of two quantum bits, it is described as tensor product. For example, in the Dirac notation, a two-quantum system is given as

$$|\psi_1\psi_2\rangle = \alpha\gamma|00\rangle + \alpha\delta|01\rangle + \beta\gamma|10\rangle + \beta\delta|11\rangle.$$

The number of possible states of the combined system increases exponentially when a quantum bit is added. This fact results in the problem of estimating the quantum correlation, which occurs for quantum bits in a compound system (see example 4).

In quantum mechanics, a quantum state $|\psi\rangle$ is expressed by the operator of density of a state ρ. The density matrix ρ of a quantum system has the following properties: $\rho^\dagger = \rho$ (Hermitian matrix); $\rho > 0$ (positive matrix); and $Tr\rho = 1$ (normalized matrix). If the state of the quantum system is known exactly, then the system is described by the density operator in the matrix form $\rho = |\psi\rangle\langle\psi|$ and is in a *pure* state. Otherwise, the system is in a *mixed* state. In this case, we have a mix of different pure states described by the density operator $\rho = \sum_i p_i|\psi_i\rangle\langle\psi_i|$ n the ensemble $\{p_i|\psi_i\rangle\}$. The matrices that satisfy the listed conditions generate a convex set. Therefore, they can be written in the form

$$\rho = \sum_k p_k|\psi_k\rangle\langle\psi_k|,$$

where $|\psi_k\rangle$ are unit vectors of the Hilbert space and $p_k > 0$, $\sum_k p_k = 1$. The coefficient p_k for a given k can be interpreted as the probability of the event that the quantum system is in a pure state $|\psi_k\rangle$.

However, this physical interpretation depends on the representation

$$\rho = \sum_k p_k|\psi_k\rangle\langle\psi_k|,$$

which is not unique—different states may have the same density matrix. In particular, vectors may be taken orthonormal and interpreted as eigenvectors of the density matrix ρ with eigenvalues p_k, and the form

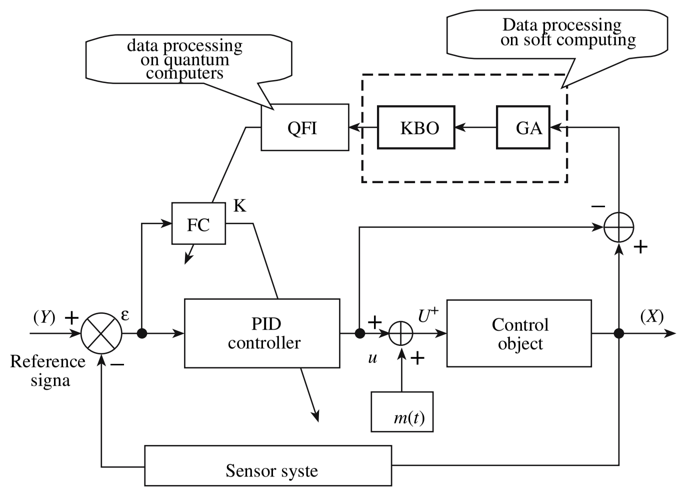
$$\rho = \sum_k p_k|\psi_k\rangle\langle\psi_k|$$

is called the spectral representation of ρ. Pure states are one-dimensional projectors $|\psi\rangle\langle\psi|$ and are extremal points of the convex set of density matrices. Thus, based on density matrices of pure states, all the other density matrices corresponding to mixed states as a convex combination in the form

$$\rho = \sum_k p_k|\psi_k\rangle\langle\psi_k|,$$

are reproduced, but they do not admit themselves a description in the form of a nontrivial convex combina-

The structure and qualitative characteristics of QFI-based self-organizing ICS

Structure of control system	Levels of control quality	Advantages and disadvantages	Limit capabilities
	Stability Controllability Control precision Adaptation Learning Self-organization Elements of self-development	A new level of control quality was introduced (self-organization with elements of self-development). Guarantees the control quality in various spaces of search for solutions of quantum fuzzy inference in on-line. The process of designing a unified KB is performed automatically by the operators of superposition and quantum interference with the help of a wise controller based on the principle of minimum of information entropy and maximum of quantum correlation (maximum of quantum amplitude of the probability of a quantum state)	Guarantees only necessary conditions for optimization of the process of designing a robust KB. In QFI, there is no possibility of optimal control of quantum operators. The process of global optimization depends on the choice of a fitness function and is performed by a combinatorial method. Requires a large amount of computational time (high temporal complexity of computations)

tion of other matrices. There is a simple criterion for determining the type of state of a quantum system: if the trace of the density matrix $Tr(\rho^2) = 1$, then the quantum system is in a pure state; if $Tr(\rho^2) < 1$, then a mixed state takes place. The definition and calculation of the trace of the density operator is given in what follows.

From the point of view of information theory, the quantum bit contains the same amount of information as the classical bit, despite an infinite set of virtual states of the quantum bit. The quantum bit can be described by an infinite number of superpositions of classical states, but because of the irreversible character of the measurement process, it is possible to extract only a simple classical bit of information from a single state among the possible ones. Note that the other virtual states are destroyed, and information is lost. The ground for this statement (a quantum bit contains no greater information than the classical one) is the fact that information is extracted as a result of a physical process. For the sake of measurement of a quantum bit, its state is changed and, as a result, it passes to one of the possible basis states. Each quantum bit exists in a two-dimensional space, its measurement is associated with the corresponding basis and expresses the result only in one of the two states; i.e., one of the basis vectors is associated with a given measurement device. Thus, as in the classical case, in a measurement of a quantum bit, there are only two possible results. Since the measurement measures a state of the quantum bit, it is impossible to register states in two different bases simultaneously. In simulating a classical dynamic system, its state can be measured at the first stage in one basis, then, at the second stage, it can be measured in another basis. In a true quantum system, this is impossible since the wave function, describing the state of the quantum bit, is destroyed in measurement. Moreover, quantum states in a true quantum system cannot be cloned; i.e., there are objective physical limitations in view of which we cannot conduct the measurement in two ways, using, e.g., copying a quantum bit and its registration in different bases [12]. In contrast to the quantum bit, the state of a classical bit can be copied and we are able to measure in different computational bases. In addition, the unknown quantum bit cannot be split into complementing parts [13]; i.e., the information that is contained in an unknown state of a quantum bit is indivisible.

Thus, in quantum mechanics, operations that are impossible in the classical mechanics are admissible, and vice versa, in classical mechanics, there are operators for solving problems that are inadmissible in quantum mechanics.

Remark 2. Computational basis $\{|+\rangle, |-\rangle\}$. To describe and measure a quantum bit, we used above the computational basis $\{|0\rangle, |1\rangle\}$. However, this choice of the computational basis is not unique. It is possible to use various sets of vectors as orthonormal bases. For example, it is admissible to represent basis vectors in the form of states $\{|+\rangle, |-\rangle\}$ defined as $\left\{\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle), \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)\right\}$, respectively. Using this representation of basis vectors, we can pass to the conventional basis

$$|0\rangle = \frac{1}{\sqrt{2}}(|+\rangle + |-\rangle) \text{ и } |1\rangle = \frac{1}{\sqrt{2}}(|+\rangle - |-\rangle).$$

Example 2. Formation of the superposition state using Hadamard operator (Walsh–Hadamard). The existence of the state of superposition and effect of measurement of a quantum state (see example 6) physically means that there is *information hidden from the observer*, which is contained in a closed quantum system (before the moment of its excitation from an external perturbation) in the form of observation of a quantum state. The system remains to be closed up to an interaction with the external environment (i.e., up to the action of system observation. The following question is the most important: How can we use efficiently information hidden in superposition (see in what follows examples in Section 3 and the results of simulation in Section 8)? In the conventional formalism of quantum computations, quantum operators are described in an equivalent matrix form. The multiplication of the matrix of an operator by a state vector means the action of the operation on the investigated system.

For example, the action of the Hadamard matrix (H) on the system $|\psi\rangle = |0\rangle$ can be represented as

$$H|\psi\rangle = H|0\rangle = \frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\1 & -1\end{bmatrix}\begin{bmatrix}1\\0\end{bmatrix} = \frac{1}{\sqrt{2}}\begin{bmatrix}1\\1\end{bmatrix}$$

$$= \frac{1}{\sqrt{2}}\left(\begin{bmatrix}1\\0\end{bmatrix} + \begin{bmatrix}0\\1\end{bmatrix}\right) = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle).$$

Similarly,

$$H|\psi\rangle = H|1\rangle = \frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\1 & -1\end{bmatrix}\begin{bmatrix}0\\1\end{bmatrix} = \frac{1}{\sqrt{2}}\begin{bmatrix}1\\-1\end{bmatrix}$$

$$= \frac{1}{\sqrt{2}}\left(\begin{bmatrix}1\\0\end{bmatrix} + \begin{bmatrix}0\\-1\end{bmatrix}\right) = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle),$$

i.e., the Hadamard transform generates a state of the quantum bit in the form of superposition of two classical states. The formation of the superposition with equivalent amplitudes of probabilities is an important step for many quantum algorithms. Applying $H^{\otimes n}$ in the corresponding basis states $|x\rangle \in \mathcal{H}_n$, $x \in \{0, 1\}^n$, we obtain as a result the equivalent form of the Hadamard transform

$$H|x\rangle = \frac{1}{\sqrt{2}} \sum_{z=0,1} (-1)^{xz}|z\rangle,$$

where $xz = x_1z_1 + \ldots + x_nz_n$ for $x = 0$ and $x = 1$. Thus, the superposition with equivalent probability amplitudes $\frac{1}{\sqrt{2^n}}$ for each basis state is obtained by the application of the operator $H^{\otimes n}$ to the state $|0\rangle$.

The value of the superposition state for the theory of computational processes becomes more understandable if we interpret the resulting superposition state as a set of 2^n classical trajectories (paths) of computations with equivalent weights, based on which the quantum computer physically conducts computations in parallel. In this sense, superposition plays the role of the first stage in order to organize *quantum parallelism.*

Example 3. Quantum parallelism and models of computations with a quantum oracle. The considered effect is one of the most important in quantum computing and is used (together with superposition) in many models of the quantum algorithms. It is especially widely applied in various models of "black box" or "quantum oracle" in constructing quantum algorithms of different classes [8–10, 14–19], e.g., for calculating functions of the following form: $g : \{0, 1\}^n \longrightarrow \{0, 1\}^m$. Since the mapping $x \longrightarrow (x, g,(x)), x \in \{0, 1\}^n$ is invertible, there is a unitary transformation U_g, which is simulated efficiently by classical computations $(x, g(x))$, so that $|x, y\rangle \longrightarrow |x, y \oplus g(y)\rangle$ for some $y \in \{0, 1\}^m$. Note that additional quantum bits necessary for implementing invertible schematic transformations are not considered here. The transformation U_f that describes the black box (as a particular case U_g) is a unitary transformation in the form of a Boolean function $f : \{0, 1\}^n \longrightarrow \{0, 1\}$. If $|y\rangle$ is the initial state $|0\rangle$, then after application of the transformation U_f, the output of the transformation $f(x)$ is $|x, f(x)\rangle$.

The physical meaning of quantum parallelism is in the presence of the effect of parallelism of computations after using the transformation U_f for the superposition state representing different values x. For example, applying U_f to the state

$$|x, y\rangle = |\psi, 0\rangle, \quad |\psi\rangle = H^{\otimes n}|0\rangle = \frac{1}{\sqrt{2^n}} \sum_{z \in \{0,1\}^n} |z\rangle,$$

we have as a result

$$U_f|\psi, 0\rangle = \frac{1}{\sqrt{2^n}} \sum_{z \in \{0,1\}^n} |z, f(z)\rangle,$$

i.e., the superposition of all possible values of the computed function.

Thus, the estimation of values of the function $f(x)$ of only one iteration step is sufficient to compute in parallel the values of $f(x)$ from all possible input arguments x. This effect is equivalent to the application of properties of the black box (single use of the internal quantum scheme). However, in reality, only one value of the function $f(x)$ is accessible in the measurement of the result of computing $f(x)$ in the superposition of all possible states, since, because of the effect of destruction of states in the superposition, only one state randomly measured is available. The discussion of the choice of a model of a quantum oracle for quantum fuzzy computing and its substantiation are given in Section 6 in what follows.

Remark 3. On computing function values with the help of the phase. The application of the operator U_f to a controlled quantum bit in the superposition state $|\varphi\rangle := \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$ yields the following result:

$$U_f|x, \varphi\rangle = \frac{1}{\sqrt{2}}(|x, f(x)\rangle - |x, 1 \oplus f(x)\rangle)$$

$$= (-1)^{f(x)}|x\rangle\frac{1}{\sqrt{2}}(|0\rangle - |1\rangle) = (-1)^{f(x)}|x, \varphi\rangle.$$

Omitting $|\varphi\rangle$, we can define a new transformation of the type $V_f : |x\rangle \longrightarrow (-1)^{f(x)}|x\rangle$. Therefore, V_f allows one to compute values of the function $f(x)$ in terms of the phase. In other words, the computation is possible through the transformation of values of f from the basis states to amplitudes relative to given basic states. Now, applying V_f to the vector $|\psi\rangle$, we obtain the following state:

$$|\psi'\rangle = V_f|\psi\rangle = \frac{1}{\sqrt{2^n}} \sum_{z \in \{0,1\}^n} (-1)^{f(x)}|z\rangle \qquad (2.1)$$

This corresponds to the application to the controlled quantum bit of the operation

$$[U_f H^{\oplus(n+1)}(|0\rangle^{\oplus n}|1\rangle)].$$

Example 4. Simulation of quantum correlation (entanglement) with the help of CNOT-similar operators. A correct estimation of the power of quantum computing is possible only with the help of finding correlations between the values of variables in the quantum algorithm at different time instants. As an example, we consider a system of two quantum bits A and B. In accordance with the law of tensor product of vector spaces, the dimension of the space $\mathcal{H}_{AB}$ of the compound system $\mathcal{H}_A$ is determined as the product of dimensions of the spaces $\mathcal{H}_B$ and $\mathcal{H}_{AB} = \mathcal{H}_A \otimes \mathcal{H}_B$; i.e., as $\{|00\rangle, |01\rangle, |10\rangle, |11\rangle\}$. The computational basis of the states for the compound system AB is expressed in terms of the basis states of the systems A and B $\{|0\rangle, |1\rangle\}$ by tensor product $|x_1, x_2\rangle = |x_1\rangle \otimes |x_2\rangle \; \forall (x_1, x_2) \in \{0, 1\}^2$. Note that in the compound system of quantum bits, there are states that cannot be expressed in terms of tensor product of particular components of quantum bits. This property is called *entanglement* or *nonseparability* of quantum states.

Assume that $|\psi_{AB}\rangle$ consists of two states. If there are two states $|\varphi_A\rangle$ in $\mathcal{H}_A$ and $|\varphi_B\rangle$ in $\mathcal{H}_B$ such that $|\psi_{AB}\rangle = |\varphi_A\rangle \otimes |\varphi_B\rangle$, then the state is called *unentangled*. Otherwise, it is an *entanglement* or is *unseparable* [8, 10, 20]. As examples, we can present the system of two quantum bits known as the Bell state of the EPR-state (in honor of the pioneers of these examples Bell, Einstein, Podolsky, and Rosen)

$$|\varphi^{\pm}\rangle = \frac{1}{\sqrt{2}}(|00\rangle \pm |11\rangle); \quad |\psi^{\pm}\rangle = \frac{1}{\sqrt{2}}(|01\rangle \pm |10\rangle).$$

When these states are measured as subsystems in a compound system of entangled states, for a known result about a state of the subsystem, we can determine exactly the state of the other subsystem (without conducting measurements on it). Therefore, there exists another variant in the interpretation of these states. If we consider one of the states as entangled, then this means that the state cannot be factorized in the state of the product of the subsystems of two states. Thus, if an operator is applied to one of the components of the entangled state, then the result of the action is not factorized over the other components, but is calculated for these components directly based on one of the measured components.

Acting by the Hadamard operator H in the Bell state $|\phi^{+}\rangle$ on the first component, we obtain as a result

$$H_1\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle) = \frac{1}{\sqrt{2}}\{(H_1|0\rangle)|0\rangle + (H_1|1\rangle)|1\rangle\}$$
$$= \frac{1}{2}(|00\rangle + |10\rangle + |01\rangle - |11\rangle),$$

or in the matrix form

$$H_1\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$
$$= \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \end{pmatrix}\left(\frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}\right) = \frac{1}{2}\begin{bmatrix} 1 \\ 1 \\ 1 \\ -1 \end{bmatrix},$$

i.e., we have a new entangled state, which is used in the feedback of the quantum genetic search algorithm for enriching the quantum correlation of the entangled state (see Fig. 1). Successively acting on the second component of the obtained state by the Hadamard operator H_2, we arrive at

$$H_2H_1\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle) = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle).$$

Regrettably, the complexity of implementation of operations by quantum gates in this approach grows with the number of entangled states connected with the controlled bit.

Remark 4. Efficient simulation of the quantum algorithm on conventional computers. Entangled states in quantum computing are regarded as an additional physical resource that allows one to increase essentially the design power compared with the conventional models of computations. The number of parameters necessary for describing non-entangled (pure) states in the given Hilbert space $\mathcal{H}_n$ (represented as the tensor product of quantum bits) increases only linearly when the number n of quantum bits grows. However, to describe the general form of the state (non-entangled or entangled), an exponential number (2^n) vector coefficients are required. Therefore, the problem of the physical resource of quantum computing cannot easily be solved.

This problem was addressed in detail from the general positions of the theory of quantum computing in [21]. In particular, it was shown that for the quantum algorithm (operating with pure states), to improve the efficiency compared with the classical analogues when the dimension the input quantum bits increases, an unlimited number of entangled states are required. Moreover, the quantum algorithm can be simulated by the conventional tools (classical algorithms) efficiently only under the presence of small amount of quantum correlation and a fixed level of tolerance of computational operations in the quantum algorithm. Independently, in [22] it was shown how to simulate the quantum algorithm with a relatively small quantum correlation by classical algorithms efficiently. The computational cost increases linearly with the number of input quantum bits, and it grows exponentially with the increase of the required amount of quantum correlation. The independent generalization of this approach was presented in [23] and the corresponding software–hardware toolkit for efficient simulation of the quantum algorithm on conventional computers was developed. This approach is harnessed in this paper for simulation in on-line of robust knowledge bases for intelligent fuzzy controllers.

The presented arguments and results confirm the preferable role of quantum correlation as a driving force of quantum computing (on pure states of the evolution of quantum dynamics).

Example 5. Simulation of quantum interference with the help of the Hadamard transform and the quantum Fourier transform (QFT). To increase the probability of measurement and extraction of the desired (marked) solution, the main unified idea in the processes of designing various models of the quantum algorithm is to use the phenomenon of constructive/destructive interference as a tool of extracting results of efficient computations of the quantum algorithm. To increase the probability of extracting a "successful" solution, the

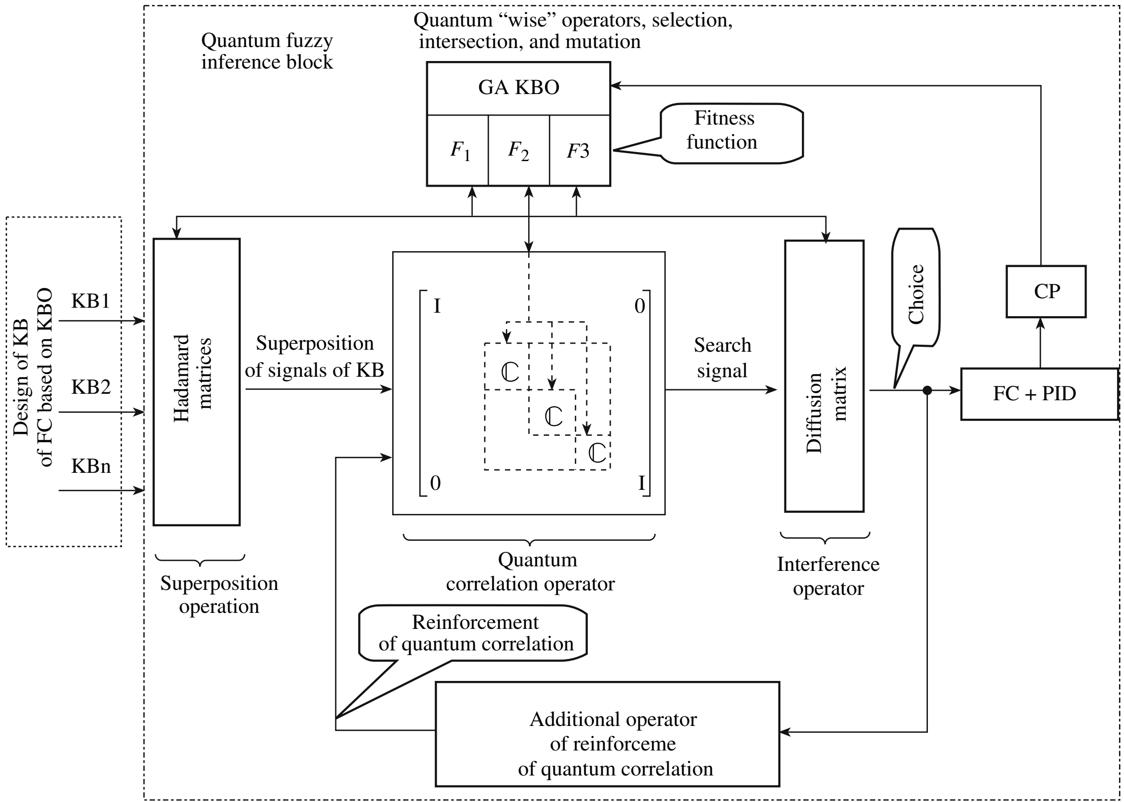


Fig. 1. The structure of the generalized quantum block.

constructive interference is applied, and to reduce "poor" solutions the destructive interference is used. The constructive (destructive) effect can be illustrated clearly by the example of application the Hadamard transform to the states$\{|0\rangle, |1\rangle\}$, and $\left\{|\pm\rangle = \frac{1}{\sqrt{2}}(|0\rangle \pm |1\rangle)\right\}$. It is obvious that $H|0\rangle = |+\rangle$ and $H|1\rangle = |-\rangle$; i.e., the state of superposition of classical states is reproduced in the form of quantum bits. Note that the application of the Hadamard transform to the states $|0\rangle$ and $|1\rangle$ generates states with the same probability distribution. Since the state $|+\rangle$ is the superposition of both classical states $|0\rangle$ and $|1\rangle$, under repeated application of the Hadamard transform to $|+\rangle$, the classical model of logical inference (Kolmogorov model) presumes the same probability of the resulting classical state (the principle of probability conservation). However, because of operation in quantum computing with the concept of amplitude of probabilities [24, 25], the application of the Hadamard transform to the state $|+\rangle$ yields the following result: $H|+\rangle = \frac{1}{\sqrt{2}}(|+\rangle + |-\rangle) = |0\rangle$.

Thus, the *interference* effect between the probabilities of both classical states becomes apparent. On the one hand, the interference (in view of its physical character) reinforced the probability amplitude of a classical state $|0\rangle$ (*constructive* interference) and weakened essentially (up to zero) the probability amplitude of the other classical state $|1\rangle$ (*destructive* interference). Acting on the superposition of possible solutions, interference implements the process of forming the final phase of quantum computing and is (as well as quantum correlation) a physical resource for reinforcing quantum computing, as well as for solving various problems of designing models of quantum algorithm. For example, applying the transform $H^{\otimes n}$ to the state $|\psi'\rangle$ in form (2.1), we obtain as a result the quantum state

$$\frac{1}{2^n} \sum_{z \in \{0,1\}^n} \sum_{x \in \{0,1\}^n} (-1)^{xz + f(x)} |z\rangle,$$

which provides a background for designing a quantum gate, e.g., in solving the Deutsch–Jozsa problem [10].

In the Shor model of quantum algorithm, in factorizing a product into prime numbers, the quantum Fourier transform provides interference: the operator $QFT_n \otimes I_n$ acts on each basis vector belonging to a linear combination of the initial vector $|\psi\rangle$. This means that any vector in this combination reproduces the superposition of base vectors. The complex weighting coefficients of the basis vectors are equal in modulus (i.e., the amplitudes of probabilities are equal), but have different phases. Each basis vector is a weighted sum of probability amplitudes obtained from different sequences of basis vectors. This sum can increase or decrease the resulting probability amplitude.

Since this effect is similar to the effect of interference of classical waves, it is said that the operator

$$[QFT_n]_{ij} = \frac{1}{\sqrt{2^n}}\exp\left\{2\pi J\left[\frac{(i-1)(j-1)}{2^n}\right]\right\}$$

plays the role of the interference operator. From the mathematical point of view, when the operator $QFT_n \otimes I_n$ acts on a state, all columns of the resulting matrix are employed in computing, and the interference is performed between the weighting coefficients from different sequences of basis vectors [26]. Consider the specific features of a very important (and frequently addressed in papers on the description of the foundations of quantum mechanics) quantum operator describing the irreversible process of extracting the result of quantum computing.

Example 6. Measurements in different computational bases. The postulate of quantum measurement was introduced by von Neumann. Only projective measurements were considered, in which "standard" quantum observable quantities A have a spectral representation in terms of orthogonal projective operators. The postulate states that, at the time of measurement of A, the state vector of the quantum system is reduced to an eigenvector of the observable quantity A corresponding to the measurement result.

Thus, the basis of the main approach to designing models of measurements in quantum computing is provided by the postulate of von Neumann projections: the result of the action of an observation of a state ψ over an observable quantity A as a result of measurement is one of the eigenvalues of A; i.e., a measurement destroys the state ψ and renormalize it. For a finite-dimensional Hilbert space, this means mathematically the following: let $\mathcal{H}$ be a Hilbert space of dimension n of a quantum system S. Assume that

$$A = \sum_{i=1}^{n}\lambda_i|\phi_i\rangle\langle\phi_i| = \sum_{i=1}^{n}\lambda_i P_i$$

is the spectral representation of the observable quantity A, where λ_i are eigenvalues and ϕ_i is the eigenvector corresponding to λ_i; $\{\phi_i\}_{i=1}^{n}$ is an orthonormal basis of the space $\mathcal{H}$ and P_i is the projection on the proper space of the operator A (spanned on the eigenvectors) with the eigenvalues λ_i. A possible value of the projective measurement corresponds to an eigenvalue λ_i of the observable quantity A. The observable quantity A of the state $|\psi\rangle = \sum_{i=1}^{n}|\phi_i\rangle$ is reproduced as the value λ_i with the probability $\left|\sum_{j=1}^{k}c_{i_j}\right|^2$, where $c_{i_1}, \ldots, c_{i_k}$ are such that $\lambda_{i_1} = \lambda_{i_2} = \ldots = \lambda_{i_{k-1}} = \lambda_{i_k} = \lambda_i$, and the system state after the measurement is defined as $\sum_{j=1}^{k}\lambda'_{i_j}|\phi_{i_j}\rangle\langle\phi_{i_j}|$, where $\lambda'_{i_j} = \frac{\lambda_{i_j}}{\sqrt{\sum|c_{i_j}|^2}}$. Thus, before measurement, for the quantum system in the state $|\psi\rangle$, a possible measurement result is defined as $p(m) = \langle\psi|P_m|\psi\rangle$, and after measurement, the system is renormalized in the state $|\psi'\rangle = \frac{P_m|\psi\rangle}{\sqrt{p(m)}}$. The completeness of equations is established by the fact from probability theory

$$1 = \sum_m p(m) = \sum_m \langle\psi|P_m|\psi\rangle. \quad (2.2)$$

The postulate on projective measurements has been developed in different directions.

Postulate of a generalized quantum measurement. The model of a generalized quantum measurement (a closed quantum system S in a finite-dimensional state space) is described by a set $\{M_m\}$ of measurement operators on the Hilbert space $\mathcal{H}$ of the quantum system S, where the subscript m specifies the possible result of the measurement process. Measurement operators satisfy the completeness condition: $\sum_m M_m^{\mathrm{T}} M_m = I$. If the state of the system S is $|\psi\rangle$ before measurement, then the probability to receive the output value m is determined as $p(m) = \langle\psi|M_m^{\mathrm{T}}M_m|\psi\rangle$.. In this expression and in what follows, the symbol "T" means the conjugation operation for a unitary operator. After measurement, the system S is renormalized as

$$|\psi'\rangle = \frac{M_m|\psi\rangle}{\sqrt{p(m)}}.$$

From this measurement model, models of projective measurements and positive operator-valued (POV) measurement measures follow as a particular case. For example if M_m satisfies two additional constraints, $M_m = M_m^{\mathrm{T}}$ and $M_m M_{m'} = \delta_{mm'}M_m$, , then we obtain the model of projective measurements presented above. If the condition $E = M_m^{\mathrm{T}}M_m$ holds, we have positive operator-valued measurement measures.

In quantum information theory, there are strict rules and laws describing processes of information extraction from an unknown quantum state. The result of projective measurements of quantum bits has to be formulated in classical terms. More precisely, any projective measurement of a quantum bit yields only one classical information bit. Therefore, despite the existence of an infinite set of possible quantum states of a quantum bit, these states are indistinguishable. There are no measurement processes in the framework of the von Neumann model with the help of which we can extract more information than a single expected information bit from the quantum bit. The identification of the state of the quantum bit is not complete; i.e., for an unknown state of the quantum bit $|\psi\rangle$, it is impossible to determine its true state using projective measurements. The measurement of the state of the quantum bit $|\psi\rangle = a|0\rangle + b|1\rangle$ corresponding to the observation $\{\mathbb{C}_1, \mathbb{C}_2\}$ (where $\mathbb{C}_1(\mathbb{C}_2)$ is the subspace spanned on the state $|0\rangle$, $(|1\rangle)$ or, in other words, in accordance with the standard computational basis $\{|0\rangle, (|1\rangle\}$) has the bit 0 (1) with the probability $|a|^2$ $(|b^2)$ as the output result, and the state $|\psi\rangle = a|0\rangle + b|1\rangle$ collapses to the state $|0\rangle$ $(|1\rangle)$. The whole other information that belongs to the superposition is irreversibly lost. Therefore for an observer, the quantum bit is represented as a random variable with a certain probability distribution. However, the quantum bit $|\psi\rangle = a|0\rangle + b|1\rangle$ can also be measured relative to other computational bases in an infinite number of ways. For example, the dual computational basis is used frequently

$$\mathcal{D}$$

$$=\left\{|+\rangle \equiv |0'\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle), |-\rangle \equiv |1'\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)\right\},$$

$$|0\rangle = \frac{1}{\sqrt{2}}(|0'\rangle + |1'\rangle), |1\rangle = \frac{1}{\sqrt{2}}(|0'\rangle - |1'\rangle).$$

Then

$$|\psi\rangle = \frac{1}{\sqrt{2}}([a+b]|0'\rangle + [a-b]|1'\rangle)$$

and the measurement $|\psi\rangle$ in this basis yields 0 (or 1) with the probability

$$\frac{1}{2}|a+b|^2 \left(\text{or } \frac{1}{2}|a-b|^2\right).$$

Remark 5. Quantization of classical operators in different computational bases (on the correspondence between the quantum and classical operators). Consider as an example the problem of ìquantizationî of a given classical operator. Assume that a single-bit negation operation (NOT gate) is the classical operator, which converts the bit (a) into its complement (2), $(1 - a)$, $(a = 0, 1)$. It is not difficult to show that, as the quantum component of this operation, it is sufficient to choose the unitary Pauli matrix of type $\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$. However, if we take the negation operation for the quantum bit in the form of the matrix $\gamma_a = |a\rangle\langle a|$, where $|a\rangle = \begin{pmatrix} 1-a \\ a \end{pmatrix}$, , then the relation $\gamma_a \xrightarrow{\sigma_x} \gamma_{1-a}$ holds exactly. On the other hand, the Pauli matrix $\sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ also implements the negation operation "NOT" $\eta_a \xrightarrow{\sigma_z} \eta_{1-a}$ under the condition that the computational basis is chosen in another way, i.e., in the form $\eta_a = |a\rangle' '\langle a|$, where $|a\rangle' = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ (-1)^a \end{pmatrix}$. . This simple example shows the dependence of the procedure of establishing the quantum-logical correspondence on the choice of states in the computational basis. Moreover, even if a computational basis is chosen, there is a set of variants for further description of the operation. For example, if an operation γ_a is chosen, then the operator $\tilde{\sigma}_x = \begin{pmatrix} 0 & e^{i\theta} \\ e^{i\theta} & 0 \end{pmatrix}$ also implements the bit conversion $\gamma_a \xrightarrow{\tilde{\sigma}_x} \gamma_{1-a}$ for given values of the angles (θ, ϕ). The standard negation operation for $\theta = \phi = 0$ is only a variant, but not the uniquely possible one. This fact is explained by the physical nature of quantum states, which are described by rays rather than vectors in the Hilbert space. The presented arguments take place even in establishing the reverse correspondence between quantum and classical operators; i.e., different classical operators can correspond to a quantum operator. For example, both the identity operation $\gamma_a \xrightarrow{\sigma_z} \gamma_a$ and the negation operation $\eta_a \xrightarrow{\sigma_z} \eta_{1-a}$ correspond to the quantum operator σ_z.

Consider the example of application of measurement models in quantum computing. Assume that a compound quantum system of two quantum bits is given in the form of the state vector in the complex state $\mathbb{C}^4$ in the computational basis

$$|\psi\rangle = a_0|00\rangle + a_1|01\rangle + a_2|10\rangle + a_3|11\rangle. \quad (2.3)$$

Note that (2.3) is a generalized entangled state. If the first bit is measured in the state $|\psi\rangle$, then there exist the following two possible results: the first bit is equal to zero $(m = 0)$ or it equals one $(m = 1)$. In the first case, the corresponding operator of generalized measurement is determined as $M_0 = |00\rangle\langle 00| + |01\rangle\langle 01|$. In the second case, $M_1 = |10\rangle\langle 10| + |11\rangle\langle 11|$. The probability of the event that the first bit in the state $|\psi\rangle$ is zero is calculated in the form $p(0) = \langle\psi|M_0^\dagger M_0|\psi\rangle = |a_0|^2 + |a_1|^2$. After measurement, the state is determined as

$$|\psi'\rangle = \frac{M_0|\psi\rangle}{\sqrt{p(0)}} = \frac{M_0|\psi\rangle}{\sqrt{\langle\psi|M_0^{\mathrm{T}}M_0|\psi\rangle}} = \frac{a_0|00\rangle + a_1|01\rangle}{\sqrt{|a_0|^2 + |a_1|^2}}.$$

In the second case, the probability that the first bit in the state $|\psi\rangle$ is one is determined as $p(1) = \langle\psi|M_1^{\mathrm{T}}M_1|\psi\rangle = |a_2|^2 + |a_3|^2$, and, after a measurement, the system state is described in the following form:

$$|\psi''\rangle = \frac{M_1|\psi\rangle}{\sqrt{p(1)}} = \frac{M_1|\psi\rangle}{\sqrt{\langle\psi|M_1^{\mathrm{T}}M_1|\psi\rangle}} = \frac{a_2|01\rangle + a_3|11\rangle}{\sqrt{|a_2|^2 + |a_3|^2}}.$$

For the considered example, the operator of generalized measurement M_m can be expressed in terms of the operators of projective measurement $M_0 = P_{00} + P_{01}$ and $M_1 = P_{10} + P_{11}$, where P_{00}, P_{01}, P_{10}, and P_{11} are the corresponding projections on the space $\mathbb{C}^4$

$$P_{00} = \begin{pmatrix} 1&0&0&0\\0&0&0&0\\0&0&0&0\\0&0&0&0 \end{pmatrix}, \quad P_{01} = \begin{pmatrix} 0&0&0&0\\0&1&0&0\\0&0&0&0\\0&0&0&0 \end{pmatrix},$$

$$P_{10} = \begin{pmatrix} 0&0&0&0\\0&0&0&0\\0&0&1&0\\0&0&0&0 \end{pmatrix}, \quad P_{11} = \begin{pmatrix} 0&0&0&0\\0&0&0&0\\0&0&0&0\\0&0&0&1 \end{pmatrix},$$

where $P_{00} + P_{01} + P_{10} + P_{11} = I^{(4)}$ and $I^{(4)}$ is the identical operator in $\mathbb{C}^4$. The matrices M_0 and M_1 are two-dimensional projectors in $\mathbb{C}^2 \otimes \mathbb{C}^2$ and can be rewritten in the form of block matrices

$$M_0 = \begin{pmatrix} I^{(2)} & 0\\ 0 & 0 \end{pmatrix}, \quad M_1 = \begin{pmatrix} 0 & 0\\ 0 & I^{(2)} \end{pmatrix},$$

where $I^{(2)}$ is the identical operator in the complex space $\mathbb{C}^2$. It can easily be tested by direct calculation that the following necessary condition holds:

$$M_0^{\mathrm{T}}M_0 + M_1^{\mathrm{T}}M_1 = I^{(4)}.$$

Consider the corresponding interrelation with positive operator-valued measurement measures, In this case for two positive definite operator-valued measures, by the definition, the relations

$$E_0 = M_0^{\mathrm{T}}M_0 = M_0, \quad E_1 = M_1^{\mathrm{T}}M_1 = M_1,$$
$$\text{and } E_0 + E_1 = I^{(4)}$$

hold. If the second bit is measured in the state $|\psi\rangle$, then there exist two other variants of possible results of measurements ($m = 2, 3$), either the second bit is zero or it equals one. In the first case the corresponding operator of generalized measurement $M_2 = |00\rangle\langle 00| + |10\rangle\langle 10|$, and for the second case, we have $M_3 = |10\rangle\langle 10| + |11\rangle\langle 11|$, where $M_2 = P_{00} + P_{10}$ and $M_3 = P_{01} + P_{11}$. It is obvious that the given measurements with possible values ($m = 2, 3$) differ from the previous variant ($m = 0, 1$).

Let us discuss the specific features of the formalism for describing quantum measurement processes by the example of generalized Bell states. We recall that the four Bell states

$$|\phi^{\pm}\rangle = \frac{1}{\sqrt{2}}(|00\rangle \pm |11\rangle); \quad |\psi^{\pm}\rangle = \frac{1}{\sqrt{2}}(|01\rangle \pm |10\rangle)$$

are maximum entangled and generate a basis of entangled states in the space $\mathbb{C}^4$. They are called "magic" frequently because of their unusual physical properties and the important role in quantum computing. Let us distinguish a particular case $|\phi^{\pm}\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$. If we measure the first bit in $|\phi^{\pm}\rangle$, then as above there exist two possible results, either zero or one. If $m = 0$, then we have the variant of the generalized measurement M_0 presented above. The probability of the event that the first bit in the state $|\phi^{+}\rangle$ is 0 is determined in the form $p(0) = \langle\phi^{+}|M_0^{\mathrm{T}}M_0|\phi^{+}\rangle = \frac{1}{2}$. After a measurement, we have the quantum state

$$|\phi^{+\prime}\rangle = \frac{M_0|\phi^{+}\rangle}{\sqrt{\langle\phi^{+}|M_0^{\mathrm{T}}M_0|\phi^{+}\rangle}} = \frac{\frac{1}{\sqrt{2}}|00\rangle}{\frac{1}{2}} = \sqrt{2}|00\rangle.$$

For the case $m = 1$, an analogue of the generalized measurement M_1 holds. The probability that the first bit in the state $|\phi^{+}\rangle$ is equal to one is calculated as $p(1) = \langle\phi^{+}|M_1^{\mathrm{T}}M_1|\phi^{+}\rangle = \frac{1}{2}$. After a measurement, we obtain the quantum state

$$|\phi^{+\prime}\rangle = \frac{M_1|\phi^{+}\rangle}{\sqrt{\langle\phi^{+}|M_1^{\mathrm{T}}M_1|\phi^{+}\rangle}} = \frac{\frac{1}{\sqrt{2}}|11\rangle}{\frac{1}{2}} = \sqrt{2}|11\rangle$$

and as a result $p(0) + p(1) = \frac{1}{2} + \frac{1}{2} = 1$ (the law of probability conservation). Note that the operators M_0 and M_1 are positive operator-valued measures for generalized observable quantities. Consider the well-known *complementarity* or *duality* principle of a particle–wave system, which is a ground for many concepts in quantum mechanics.

Quantum particle–wave duality in models of measurement processes. At the structural level, the model of quantum mechanics consists of two constituents, the first of which operates with the concept of quantum state of the investigated quantum system, and the second one is oriented to quantum dynamics (evolution of the quantum system). The presence of two approaches is connected with the particle–wave duality of the description of dynamic objects of quantum mechanics. Note certain specific features and properties of quantum evolution and the interrelation of measurements processes with the duality of the representation of dynamic objects in quantum mechanics. Consider the wave representation of quantum mechanics. Assume that the state of a quantum system is described in the following form: $|\psi\rangle = \sum_i a_i|i\rangle$, where $\sum_i a_i = 1$, and $\{|i\rangle\}$ determines the set of orthogonal vectors. If $i \geq 2$, then we say that the dynamics of the given quantum system have a wave character. Otherwise $i = 1$, the quantum system describes the dynamic behavior of a single particle, and we deal with the corpuscular behavior of the considered system.

The trace of a matrix and processes of quantum measurements.,. Let us consider the trace of a matrix A, A_{ii}: $Tr(A) \equiv \sum_i A_{ii}$. The matrix trace has the cyclic property and is linear. Suppose that the state of two physical systems A and B is described by the density operator ρ^{AB}. The reduced density operator for the system A can be written in the following form: $\rho^A \equiv Tr_B(\rho^{AB})$, where the partial trace Tr_B is the mapping of operators known as the partial trace over the system B

$$\rho^A \equiv Tr_B(\rho^{AB}) = Tr_B(\rho^{AB} = |a_1\rangle\langle a_1| \otimes |b_1\rangle\langle b_1|) \equiv$$
$$\equiv (|a_1\rangle\langle a_1|)Tr(|b_1\rangle\langle b_1|).$$

Thus, the partial trace of an operator is a tool for quantitative description of the observed subsystems belonging to a compound system.

To distinguish quantum states, quantum measurements are required. A projective measurement over a subsystem is similar to the operation of taking the partial trace. Assume that we have a state GHZ of the following form:

$$|\mathrm{GHZ}\rangle = \frac{1}{\sqrt{2}}(|0_A 0_B 0_C\rangle + |1_A 1_B 1_C\rangle)$$

(an entangled three-particle state of three quantum bits ABC). Then the partial trace over one quantum bit, e.g., over the system A, can be written as

$$Tr_A(|\mathrm{GHZ}\rangle\langle\mathrm{GHZ}|) = \frac{1}{2}(\rho_{00}^{BC}) + \frac{1}{2}(\rho_{11}^{BC}),$$

where $\rho_{00}^{BC} = |0_B 0_C\rangle\langle 0_B 0_C|$, $\rho_{11}^{BC} = |1_B 1_C\rangle\langle 1_B 1_C|$; i.e., after taking the partial trace over system A, the subsystem BC has the probability $p = \frac{1}{2}$, and is either in the state ρ_{00}^{BC} or in ρ_{11}^{BC}. According to the postulate of quantum measurements, if a projective measurement in the basis $\{|0\rangle, |01\rangle\}$ was carried out over the system A, then the result is the state $|0\rangle$ (corresponding to ρ_{00}^{BC}) or the state $|01\rangle$ (corresponding to ρ_{11}^{BC}) with the probability $p = \frac{1}{2}$.

Consider the quantum state $|w\rangle$ of three quantum bits of the form $|w\rangle = \frac{1}{\sqrt{3}}(|1_A 0_B 0_C\rangle + |0_A 1_B 0_C\rangle + |0_A 0_B 1_C\rangle)$. Let us take the trace over the quantum bit A

$$Tr_A(|w\rangle\langle w|) = \frac{1}{3}\rho_{00}^{BC} + \frac{2}{3}|\psi_{BC}^{+}\rangle\langle\psi_{BC}^{+}|,$$

where $|\psi_{BC}^{+}\rangle = \frac{1}{\sqrt{2}}(|0_B 1_C\rangle + |1_B 0_C\rangle)$ determines the Bell state. Thus after the projective measurement over the system A in the basis $\{|0\rangle, |1\rangle\}$ the system with the probability $p = \frac{1}{3}$ can be in the state $|0\rangle$ (corresponding to ρ_{00}^{BC}) and with the probability$p = \frac{2}{3}$, in the state $|1\rangle$ (corresponding to $|\psi_{BC}^{+}\rangle$).

Assume that Q is compound system (of two quantum bits Q_1 and Q_0), which is in the Bell state

$$|\Psi_Q\rangle = \frac{|0_1 0_0\rangle - |1_1 1_0\rangle}{\sqrt{2}}.$$

The density operator ρ_Q has the representation

$$\rho_Q = \left(\frac{|0_1 0_0\rangle - |1_1 1_0\rangle}{\sqrt{2}}\right)\left(\frac{\langle 0_1 0_0| - \langle 1_1 1_0|}{\sqrt{2}}\right)$$
$$= \frac{1}{2}(|0_1 0_0\rangle\langle 0_1 0_0| - |0_1 0_0\rangle\langle 1_1 1_0| \tag{2.4}$$
$$- |1_1 1_0\rangle\langle 0_1 0_0| + |1_1 1_0\rangle\langle 1_1 1_0| = \frac{1}{2}\begin{pmatrix} 1 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 1 \end{pmatrix}.$$

Let us calculate the partial trace based on the state of the second quantum bit; i.e., the system Q_1. As a result the reduced density operator $\rho_{Q_0} \equiv \rho_0$ has the following form:

$$\rho_0 = Tr_1(\rho_0)$$
$$= \frac{1}{2}[Tr_1(|0_1 0_0\rangle\langle 0_1 0_0|) - Tr_1(|0_1 0_0\rangle\langle 1_1 1_0|)$$
$$- Tr_1(|1_1 1_0\rangle\langle 0_1 0_0|) + Tr_1(|1_1 1_0\rangle\langle 1_1 1_0|)]$$
$$= \frac{1}{2}[|0_0\rangle\langle 0_0|\langle 0_1|0_1\rangle - |0_0\rangle\langle 1_0|\langle 1_1|0_1\rangle - |1_0\rangle\langle 0_0|\langle 0_1|1_1\rangle$$
$$+ |1_0\rangle\langle 1_0|\langle 1_1|1_1\rangle] = \frac{1}{2}[|0_0\rangle\langle 0_0| + |1_0\rangle\langle 1_0|]$$
$$= \frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \frac{1}{2}I.$$

Note that it was taken into account that the states are orthogonal, $\langle 0_1|0_1\rangle = 1$, $\langle 0_1|1_1\rangle = 0$, etc. Note that in this example the reduced state ρ_0 is mixed since

$$Tr[(\rho_0)^2] = Tr\left[\left(\frac{I}{2}\right)^2\right] = \frac{1}{2} < 1.$$

This result characterizes the specific feature of the quantum system that is principally absent in the classical system [8]. Indeed, the state of a compound system is pure, $Tr[(\rho_Q)^2] = 1$, and it is established exactly (maximum information). However, the first quantum bit is in a mixed state. This means that information about the state of this quantum bit is not maximal, and the system itself has more chaotic behavior than the compound system. A similar property, which strange for classical systems, when complete information about the compound system is available, and for the subsystems only a part of information is known, characterizes the ability of quantum systems to be self-organized by using the super-correlation property (quantum correlation) contained in entangled states. It is the presence of quantum correlation that explains the possibility to organize a pure state in two mixed compound states.

The Schredinger equation describes the evolution of the state of a quantum system. The generalized quantum dynamic processes are formalized by quantum operators. The complementarity principle presumes the use of particle–wave duality in describing a quantum object. To extract information on the behavior of the quantum object, it is necessary to conduct quantum measurements. Unitary operations are connected with the wave representation of a quantum object. However, non-unitary operators violate the wave pattern of system description and result in a corpuscular representation. In the quantum case, completeness and correctness of relations require that the condition $Tr(\rho) = 1$ is satisfied, which is valid for quantum measurements by condition (2.2). The trace operation means that we can detect a particle in the state space with definiteness, after which the wave behavior of the particle is destroyed.

Consider the case of quantum measurements with the help of projective operations. As was mentioned above, the probability of the measurements result is determined in the form $p(m) = \langle\psi|M_m^{\mathrm{T}} M_m|\psi\rangle$. Since the state of the quantum system is orthogonal after a projective measurement, by the definition $|\psi\rangle = \sum_i a_i|i\rangle$, $i = 1$, its behavior has the *corpuscular character*. Thus, the wave representation is destroyed. However, after positive operator-valued measurements, the quantum system may display wave properties. Assume that the quantum system is two-dimensional in the basis $\{|0\rangle, |1\rangle\}$. Consider the positive operator-valued measure of measurement in the form of three operators

$$E_1 = \frac{\sqrt{2}}{1+\sqrt{2}}|1\rangle\langle 1|;$$
$$E_2 = \frac{\sqrt{2}}{1+\sqrt{2}}\frac{1}{2}(|0\rangle - |1\rangle)(\langle 0| - \langle 1|);$$
$$E_3 = I - E_1 - E_2,$$

which are not pairwise orthogonal. After positive operator-valued measurements, the quantum system displays both the *wave* and *corpuscular* behavior. This complementarity exists between an entangled state (quantum correlation) and interference. The measure of quantum correlation (entanglement) is a decreasing function of the visibility measure depending on interference. Consider the complementarity between quantum correlation and interference by the example of the generalized Bell state.

Example 7. Complementarity between an entangled state and interference. Let us take the generalized (entangled) Bell state $|\psi\rangle = a|0_1\rangle|0_2\rangle + b|1_1\rangle|1_2\rangle$ with a limitation of the probability amplitudes following from the unitarity condition $a^2 + b^2 = 1$. Let us perform a unitary transformation with the help of the rotation operator (matrix) with respect to the first computational basis $\{|0_1\rangle, |1_1\rangle\}$ for the first quantum bit $|0_1\rangle \longrightarrow \cos\alpha|0_1\rangle + \sin\alpha\,|1_1\rangle$ and obtain as a result

$$|\psi\rangle \longrightarrow |\psi'\rangle = a(\cos\alpha|0_1\rangle + \sin\alpha|1_1\rangle)|0_2\rangle$$
$$+ b(\cos\alpha|1_1\rangle - \sin\alpha|0_1\rangle)|1_2\rangle.$$

The next operation is the observation of the first quantum bit without perturbation of the second one. The probability of extracting the state $|0_1\rangle$, determined as $P_{|0_1\rangle} = \frac{1}{2}[1 + (a^2 - b^2)\cos 2\alpha]$, is a typical example of the interference effect if the angle α is interpreted as a controlled parameter. The visibility measure of interference is $\Gamma \equiv |a^2 - b^2|$. As a physical measure of the measurement result, it disappears if the initial state was maximum entangled (maximum correlated), i.e., for

$a^2 = b^2$. This physical measure is maximal when the state is separable (i.e., for either $a = 0$ or $b = 0$).

On the other hand, it is known that the partial trace over one of the states in the form of the von Neumann entropy (see the definition in what follows)

$$E \equiv S(\rho_{\text{red}}) = -a^2 \log a^2 - b^2 \log b^2,$$

where the reduced density operator

$$\rho_{\text{red}} = Tr_2|\psi'\rangle\langle\psi'| = Tr_2|\psi\rangle\langle\psi|$$

$$a^2|0_1\rangle\langle 0_1| + b^2|1_1\rangle\langle 1_1|.$$

is the measure of entanglement of the state (quantum correlation). The quantum correlation takes the maximum value $E = 1$ for $a^2 = b^2$, and it takes the minimum value $E = 0$ if either $a = 0$ or $b = 0$. Thus, the greater is the quantum correlation of the state, the lower is the interference visibility, and vice versa.

Another popular correlation measure, called negativity N, is defined as a negative value of the doubled value of the least eigenvalue of the transposed density matrix. Therefore, $N = 2|ab|$. In this case, the complementarity measure is expressed as $N^2 + \Gamma^2 = 1$. Then, the interrelation between the entangled state and interference follows from the constraint in the form of unitarity of quantum operators $a^2 + b^2 = 1$.

Thus in models of quantum algorithm, the measures of quantum correlation and interference are not independent, and the efficiency of finding a successful solution with the help of the quantum algorithm depends on their interrelation.

Example 8. Design of quantum algorithmic gates and quantum programming. The ground of the method of designing quantum algorithmic schemes and similar methods for forming new types of quantum algorithms is provided by the system for designing quantum algorithmic gates (QAGC) described in [8, 10, 23]. As in the general structure of the quantum algorithm, the structure of the system for designing QAG (quantum algorithmic gates) is based on the formalization of the description of three main quantum operators (superposition, quantum correlation (of entangled states), and interference) and measurement in the form of elementary evolutionary unitary operators. In accordance with the quantum scheme of the QA (quantum algorithm), these operators are combined by tensor and direct products in a unified evolutionary quantum unitary operator [10, 23]. Structurally, the quantum algorithmic gates acts on the initial canonical basis vector and forms a complex linear combination of the constituent classical vectors (called superposition) in the form of basis vectors as the output result of the action of the superposition operator. The superposition contains information about the solution of the investigated problem as one of the components. After forming the superposition, in the quantum algorithmic gates, the operators of quantum correlation, interference, and measurement are applied in order to extract information about the desired solution. In quantum mechanics, the measurement process has the irreversible character and is a non-deterministic operation, which results in measurement of only one of the basis vectors in the formed superposition. The probability of each basis vector to be a measurement result in the composition of the superposition for a given computational basis depends on the complex coefficient (probability amplitude). The process of termination of the iterative action of the quantum algorithmic gates is performed by a program method based on the principle of minimal information entropy of an "intelligent quantum state", containing valuable information about the desired solution [10, 26]. quantum algorithmic gates can be implemented with the help of a software–hardware toolkit of evolutionary quantum computing.

In quantum programming, there exists a proof of completeness of the description of a quantum algorithmic gate by the corresponding programming languages of improved semantic expressiveness [10]. By analogy with the existence of equivalence in the theory of computing, which is based on classical algorithms, we know a hypothesis on equivalence between the representation of expressions of quantum operations at the syntactic level with conservation of the completeness of their description for the sake of inclusion in quantum programming languages of semantic expressiveness of quantum operators (at the functional level of the description of actions of quantum operators). One of the natural steps in this direction is the development of principles of logical inference and test of the truth of propositions in quantum programming languages for avoiding contradictions in the obtained consequence of logical inference. We refer to these language, e.g., the QML (Quantum Programming Language) [10, 27–34].

Consider how a consistent description of the definition of action of the Hadamard operator is implemented in QML in the following form:

$$Hx = \text{if } x \text{ then}(\textit{false} + (-1)\textit{true})\text{else}(\textit{false} + \textit{true}).$$

Let us estimate the completeness and truth of this expression, which is equivalent to the test of truth of the fact that the successive action of Hadamard operators $H(Hx)$ leads to the result equivalent to x at the functional level. For this purpose we use the following model of logical inference in QML [29]:

```
H(Hx) = if (if x then (false + (–1)true) else (false + true))
            then (false + (–1)true)
            else (false + true)
            – – by commuting conversion for "if"
        = if x
            then if (false + (–1)true)
                    then (false + (–1)true)
                    else (false + true)
            else if (false + true)
                    then (false + (–1)true)
                    else (false + true)
```

– – by "if"
= if x
then (*false – false + true + true*)
else (*false + false + true – true*)
– – by simplification and normalisation
= if x then *true* else *false*
– – by η-rule for "if"
= x

Elements of the theory of testing completeness and truth of the semantics of the functional description of the quantum algorithm in the QML functional quantum programming language were described in [10, 27–34].

3. QUANTUM INFORMATION PROCESSES AND LAWS OF QUANTUM INFORMATION THEORY

The Shannon information entropy is defined as

$$H(p) = -\sum_i p_i \log p_i.$$

The von Neumann entropy has the following form:

$$S^{vN}(\rho) = -Tr(\rho \log \rho).$$

In the particular case, when the matrix ρ is diagonal, the Shannon and von Neumann entropies are equal. However, the laws and consequences of quantum information theory have a number of fundamental distinctions under the quantum generalization of Shannon classical information theory [8, 10, 35–37]. Let us investigate briefly some of these specific features employed in the models of quantum fuzzy inference by examples.

Example 9. Consider the specific features of the description and information analysis of Bell entangled states

$$|\Psi_Q\rangle = \frac{|0_1 0_0\rangle - |1_1 1_0\rangle}{\sqrt{2}}.$$

Since the Bell state with density operator (2.4) is pure, ρ_Q represents a pure ensemble. Therefore, there is no uncertainty in the quantum state; i.e., the von Neumann entropy is zero $S^{vN}(\rho_Q) = 0$. The reduced density operator ρ_0 for the quantum bit $|0_0\rangle$ is the partial trace over the system Q, i.e.,

$$\rho_0 = Tr_1(\rho_Q) = \frac{1}{2}(|0\rangle\langle 0| + |1\rangle\langle 1|) = \frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}.$$

Therefore, the quantum uncertainty in the state $|0_0\rangle$ is determined by the von Neumann entropy as $S^{vN}(\rho_Q) = 1$. Thus, the information analysis of uncertainty in the compound quantum system allows one to explain clearly the presence of unusual (non-classical) properties: if we ignore a part of information about the state of a subsystem, this results in an increase of quantum uncertainty. As a result, quantum uncertainty in a part (subsystem) Q_0 is greater than in the "complete" (compound) quantum system Q. Classical systems do not have this effect in view of the properties of the Shannon information entropy.

Example 10. Consider the Bell state $|\psi\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$ in the Hilbert space $\mathcal{H}_{AB} = \mathcal{H}_A \otimes \mathcal{H}_B$, where $\mathcal{H}_A = \mathcal{H}_B = \mathcal{H}_2$. The density matrices $\rho_{AB} = |\psi\rangle\langle\psi|$, ρ_A, and $\rho_{A|B}$ determined as

$$\rho_A = \begin{pmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{pmatrix}, \quad \rho_{A|B} = \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 \end{pmatrix},$$

$$\rho_{AB} = \begin{pmatrix} \frac{1}{2} & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \frac{1}{2} & 0 & 0 & \frac{1}{2} \end{pmatrix}.$$

The matrix of conditional density is $\rho_{A|B} = \rho_{AB}(I_A \otimes \rho_B)^{-1}$ (in this case the cross density matrix ρ_{AB} and the marginal density matrix $I_A \otimes \rho_B$ commutes). It follows from the definition of the von Neumann entropy that $S^{vN}(A) = S^{vN}(B)$. Then, we have

$$S(AB) = S(B) + C(A|B) = 1 - 1 = 0,$$

since $S(A|B) = -1$.

Therefore, in contrast to the Shannon classical information theory, the von Neumann quantum conditional entropy can take *negative* values when entangled states are considered. This fact is connected directly with the quantum non-separability of entangled states, and they are interpreted as super-correlated states. Thus, the negativeness of the conditional entropy indicates the presence of entangled states in a compound quantum system and determines the lower bound of their correlation [8, 10, 38, 39].

The validity of this fact was also established in the Shor and Grover quantum search algorithms [10, 26] and is harnessed in solving the problem of efficient termination of the quantum algorithm. We also note that not all base classical relations and inequalities have quantum analogues. For example, in the classical case, we have

$$I(x:y) \le \min[H(x), H(y)].$$

while, in the quantum case, the upper bound is given by the inequality

$$S(X:Y) \le 2\min[S(X), S(Y)].$$

Quantum information theory has strictly substantiated rules of how to extract information from an unknown quantum state [8, 10, 35–37]. An optimal quantum process of extraction of valuable information from individual knowledge bases designed for fixed control situations on the basis of soft computing is based on four facts of quantum information theory presented below. In particular, it was proved that there exist an efficient quantum data compression, coupling of classical and quantum parts of information in the quantum state, and total correlation in the quantum state is a mix of classical and quantum correlations, and there is a hidden (observed) classical correlation in the quantum state. In what follows, we consider briefly the physical meaning of the listed facts and their role in the processes of designing optimal processes and control signals based of quantum fuzzy inference.

Fact 1. Efficient quantum data compression. In the classical information theory, Shannon showed how much (under given accuracy) we can compress limitedly a message comprising N independent symbols (x_a), where each symbol enters the message with the prior probability p_a, using the concept of information entropy. The Shannon information entropy $H(p_a)$ is defined as

$$H(p_a) = -\sum_a p_a \log_2 p_a.$$

The following proposition was proved: a block of codes of length NH bits is sufficient for coding all typical (most frequently occurred) sequences without account of the methods of coding non-typical sequences of the message. Note that the probability of error coding (information loss) does not exceed a given threshold ε. In quantum information theory, symbols are density matrices. Two variants are possible. In the first variant, the density matrices correspond to an ensemble of pure states $|\phi_a\rangle$. In the second variant, the ensemble is formed by density matrices ρ_a with the probability p_a. Consider the ensemble of states for the second variant. In this case, the matrix of density of messages comprising N symbols is described as $\rho^{(N)} = \rho \otimes \rho \otimes \ldots \otimes \rho$, where $\rho = \sum_a p_a |\phi_a\rangle\langle\phi_a|$.

The von Neumann entropy of messages has the simple relation with the ensemble entropy, $S = -Tr(\rho \ln \rho)$. The following inequality that relates the Shannon information entropy and the von Neumann entropy is known: $S(\rho^{(N)} = NS(\rho)$; i.e., the value of the Shannon information entropy exceeds the value of the von Neumann entropy. This means that the application of quantum information theory allows one to perform deeper compression of classical information [8, 10].

Fact 2. Coupling (separation) of information in the quantum state in the form of classical and quantum components. Consider the model of a generalized measurement (see Section 2) in the state $A_i A_i^{\mathrm{T}}$, for which the density matrix has the following definition:

$$\rho_B^i = \frac{A_i \rho_B A_i^{\mathrm{T}}}{Tr(A_i \rho_B A_i^{\mathrm{T}})}.$$

Then, the final state of the subsystem B is

$$\sum_i A_i \rho_B A_i^{\mathrm{T}} = \sum_i p_i \rho_B^i.$$

The entropy of the reduced state is $\sum_i p_i S(\rho_B^i)$. The amount of classical information received in the measurement i is expressed with the probability p_i as the Shannon information entropy $H(p)$. If the quantum states ρ_B^i belong to orthogonal subspaces, then the entropy of the final state (after a measurement) is the sum of the reduced quantum entropy $\sum_i p_i S(\rho_B^i)$ and the classical information, i.e.,

$$S\left(\sum_i p_i (\rho_B^i)\right) = \underbrace{H(p)}_{\text{Classical}} + \underbrace{\sum_i p_i S(\rho_B^i)}_{\text{Quantum}}.$$

Thus, the amount of information contained in the quantum state can be separated (coupled in the form) into the quantum and classical components [40]. Therefore, in simulating robust structures of intelligent control systems, the classical information is simulated by using the knowledge base optimizer, and its *deficit* can be defined as [8, 10]

$$\Delta I = \underbrace{S\left(\sum_i p_i \rho_B^i\right)}_{\text{Total}} - \underbrace{\sum_i p_i S(\rho_B^i)}_{\text{Quantum}} = \underbrace{H(p)}_{\text{Classical}}.$$

Therefore, we can extract additional amount of valuable quantum information from individual knowledge bases for subsequent use in designing intelligent control of improved level. Note that quantum procedures for compression and reduction of redundant information contained in classical control signals are applied (using the corresponding models of quantum correlation in the quantum algorithm of quantum fuzzy inference).

Fact 3. Amount of total, classical, and quantum correlation. Entangled states or, in the general form, quantum correlation are typical physical resources of quantum computing. However, not all types of correlation have pure quantum nature. In other words, total correlations are mixes of classical and quantum correlations [41].

For optimal design (efficiently simulated on conventional computers) of a given class of quantum algorithms, it is important to know the type (or form) of the necessary classical correlation. For example, if it is possible to determine the classical component of the correlation, then, using optimal positive operator-valued) measurement measures (see the details in Section 2), it is admissible to extract the maximum amount of information in the classical form contained in the quantum state with the minimum increasing of entropy [42, 43]. The amount of total correlation can be split into the classical and quantum components. This measure is equivalent to the measure of maximum classical/quantum mutual information $I(A : B)$, retaining the direct physical interpretation of the interrelations between the corresponding measures [44].

Fact 4. Hidden (observable) classical correlation in the quantum state. In quantum information theory, the following unexpected fact was established. The condition of proportional increase of the amount of information

$$I_{Cl}(\rho) = \max_{M_A \otimes M_B} I(A : B),$$

determined by local measurements $M_A \otimes M_B$ in the state ρ_{AB} can be violated under some extremal constraints on the initial mixed state ρ. For example, the initial amount of information in the form of a single classical information bit sent from A to B can be enlarged at the receipt stage by a definite amount in the quantitative measure $I_{Cl}(\rho)$ [45]. This fact is explained from the position of the observation phenomenon of the classical correlation in the quantum state ρ. Since the amount of information $I_{Cl}(\rho)$ is proportionally increased at the classical level, the phenomenon of correlation observation is a pure quantum effect arising due to the fact that quantum non-orthogonal states are indistinguishable. Therefore, there exist quantum two-particle states that contain a large amount of classical correlation non-observable at the classical level because of the amount of classical information in the transmission channel that is non-proportionally small for its observation (limited data transmission ability).

There are $(2n + 1)$ quantum bits, using which a single-bit message increases twice the optimal amount of classical mutual information as a result of measurements between the subsystems. In general, for $n/2$ bits sent, the specified amount of information increases to n bits. It is impossible to obtain the specified effect at the classical level because of the laws of classical physics. In this case, the following fact is remarkable: the states that support the specified effect are not necessarily entangled, and the corresponding classical data exchange channel can be implemented by the Hadamard transform. The presented facts yield the information resource of the background of quantum fuzzy inference employed in simulation of robust knowledge bases for intelligent fuzzy controllers.

4. TOTAL AND HIDDEN (OBSERVABLE) CORRELATION IN QUANTUM STATES

There is a belief that the expected computational power of quantum computing issues from the presence of quantum resource. Entangled states, or quantum correlation in the general case are bright examples of this. However, as was mentioned in Section 3, not all correlation forms have purely quantum nature; i.e., the total correlation is a mix of classical and quantum correlations. The knowledge of how and where the classical correlation is used in the quantum correlation is an important point. For example, if it is possible to determine and select the classical correlation component, then using optimal measurement, we can extract certain additional information amount in the classical form hidden in the quantum state with a minimal increasing of entropy.

Physically, the listed correlation types are characterized by the amount of work (noise) that is necessary to do for eliminating (destroying) the correlation. For the total correlation, the amount of work for complete destroying is required. For the quantum correlation, the amount of work to the destruction into separable states is sufficient. However, even in the case of classical correlation, the maximum correlation is destroyed after eliminating the quantum correlation. The total amount of correlation, measured by the minimum production of randomization and equivalent to the requirement of total destruction of all forms of correlation in the state ρ_{AB}, is equivalent to the quantum amount of mutual information [46, 47].

4.1. Classical and Quantum Correlations

The classical mutual information contained in the quantum state ρ_{AB} (before its measurement) can be estimated in a natural way as the maximum mutual information which can be extracted by local measurements $M_A \otimes M_B$ in the state ρ_{AB}

$$I_{Cl}(\rho) = \max_{M_A \otimes M_B} I(A : B).$$

Here, $I(A : B)$ is the classical mutual information defined in the form

$$I(A : B) \equiv H(p_A) + H(p_B) - H(p_{AB}),$$

where H is the information entropy and p_{AB}, p_A, and p_B are the density functions of the probability distribution of the mutual and individual results obtained by local measurements $M_A \otimes M_B$ in the state ρ [45].

Example 11. Mutual information and classical correlation. To understand the role of classical correlation and its interrelations with the concept of mutual information, we define quantum mutual information for the two-particle state ρ_{AB} of the quantum system in the Stratonovich form

$$I(A : B) = S(\rho_A) + S(\rho_B) - S(\rho_{AB}).$$

Consider a compound system AB in the state ρ_{AB}, which can be in the state ρ_A and ρ_B with the probabilities p and $(1 - p)$, respectively. For this case of the compound system AB, mutual information can be calculated in the following form [42]:

$$I(A:B) = 2H\left(\frac{1}{2}[1 + \sqrt{p^2 + (1-p)^2}]\right) - H\left(\frac{1}{2}[1 + \sqrt{1 + 3p^2 - 3p}]\right). \tag{4.1}$$

If ρ_{AB} is a separable state, then its relative entropy in the entangled state is zero.

The physical interpretation of the value $I_{Cl}(\rho)$ is many-valued [45, 48]: $I_{Cl}(\rho)$ manifests itself as maximum classical correlation extracted by a purely local measurement procedure from the state ρ; $I_{Cl}(\rho)$ corresponds the classical definition when the state ρ is classical, i.e., diagonal in some (locally used) computational basis and corresponds to the classical distribution; if ρ is a pure state, then $I_{Cl}(\rho)$ specifies the correlation determined by the Schmidt basis and equivalent to the measure of entangled pure states; $I_{Cl}(\rho) = 0$ if and only if

$$\rho_{AB} = \rho_A \otimes \rho_B.$$

It is known that some suitable measures of quantum correlation have to satisfy certain axiomatic properties: (1) quantum correlation is non-local and cannot increase in local measurement procedures (monotonicity property); (2) complete proportionality; (3) growth of proportionality; and (4) continuity in ρ.

Physically, property (2) means that the state protocol constituted of the non-correlated initial state using l quantum bits or $2l$ classical bits (for data transmission through a quantum communication channel) and applying local operations cannot generate more than $2l$ correlation bits. Property (3) presumes that in transmitting a message of l quantum bits or $2l$ quantum bits the correlation in the initial state does not increase and does not exceed $2l$ bits.

Properties (1)–(4) fulfill completely, in particular, for classical mutual information $I(A:B)$ when the message transfer is performed by the classical method. For example, the properties of the complete proportionality and growth of proportionality $I(A:B)$ for the classical case follow from the fact that

$$\max(H(p_A), H(p_B)) \le H(p_{AB}) \le H(p_A) + H(p_B),$$

so that when A sends a classical system A' to B, we have

$$I_{Cl}(\rho) = I(A;BA') \le I(AA', B) + H(p_{A'}).$$

Then the property of complete proportionality follows from the property of proportionality growth. The same is true for quantum mutual information

$$I_Q(A:B) = S(\rho_A) + S(\rho_B) - S(\rho_{AB}).$$

It was unexpected that the property of proportionality growth is violated for $I_{Cl}(\rho)$ in an extremal way in the case of mixed initial states ρ—simple classical bit sent from A to B can result in an increase of $I_{Cl}(\rho)$ to a certain big value. We consider this phenomenon as an opportunity of observation of classical correlation in the quantum state ρ. If the property of growth of proportionality $I_{Cl}(\rho)$ takes place at the classical level, then the phenomenon of observed classical correlation is a pure quantum effect. This result immediately follows from indistinguishability of non-orthogonal quantum states.

Example 12. Assume that an initial state ρ, the type of message transfer, and the corresponding amount of data transferred are given. The increase of correlation can be characterized by the following functions:

$$I_{Cl}^{(l)} = \max_{\Lambda^{(l)}} I_{Cl}(\Lambda^{(l)}(\rho))$$
(one-way classical message);

$$I_{Cl}^{[l]} = \max_{\Lambda^{[l]}} I_{Cl}(\Lambda^{[l]}(\rho))$$
(two-way classical message).

The operator Λ is an operation on a two-particle state, which consists of local operations and contains no more than $2l$ classical or l quantum bits in a message. This is reflected in the corresponding superscripts (l) or $[l]$. Denote by ρ and ρ' the states before and after the operations with message exchange, $\rho' = \Lambda(\rho)$. The amount of correlation $I_{Cl}^{(L)}(\rho)$, hidden (unobservable) in the state with l quantum bits in one-way message exchange can be bounded by the following condition [45]:

$$I_{Cl}^{(L)}(\rho) - I_{Cl}(\rho) \le l + (2^l - 1)I_{Cl}(\rho).$$

For small values $I_{Cl}(\rho)$, the amount of hidden (unobservable) correlation in the two-way exchange is bounded from above

$$I_{Cl}^{[l]}(\rho) - I_{Cl}(\rho) \le 2l + O(d^2\sqrt{I_{Cl}(\rho)}\log I_{Cl}(\rho)).$$

4.2. Hidden (Observable) Classical correlation in a Quantum State

Let us discuss the situation in which some amount of correlation is not accessible to observation in one-way message exchange. The initial state is determined by the subsystems A and B on the corresponding subspaces of dimensions $2d$ and d in the form

$$\rho = \frac{1}{2d}\sum_{k=0}^{d-1}\sum_{t=0}^{1}(|k\rangle\langle k|\otimes|t\rangle\langle t|)_A \otimes (U_t|k\rangle\langle k|U_t^{\mathrm{T}})_B, \quad (4.2)$$

where the operators $U_0 = \mathrm{I}$ and U_1 change the initial computational basis by the united one as $|\langle i|U_1|k\rangle| = \frac{1}{\sqrt{a}}$ $\forall$ i, k. Then, the player B takes randomly a state $|k\rangle$ of d states in two possible randomized bases (depending on the case when either $t = 0$ or $t = 1$ in (4.2)

At the same time, the observer A has complete information about the quantum state of the observer B. Receiving the required amount of information in the form $I_{\mathrm{Cl}}^{(l)}(\rho) = \log d + 1$, A sends a value t to B, which, in turn, applies the operator U_i to its state and measure the value k in the computational basis. As a result, A and B have the measurement k and t, which yields $I_{\mathrm{Cl}}^{(l)}(\rho) = \log d + 1$ correlation bits.

The state ρ evolutes according to the following scenario [45]. Let $d = 2^n$. Then, A chooses randomly k of length n bits and sends to B either a message about the state $|k\rangle$ or $H^{\otimes n}|k\rangle$ depending on a random value of the bit either $t = 0$ or $t = 1$. Here, H is the Hadamard transform; A sends t to B and, later, observes the created correlation. It was experimentally established that the application of the Hadamard transform and measurement of the state of the quantum bits is sufficient in order to implement the procedure of preparing the state ρ and then extract the classical correlation hidden in the state ρ'. The initial correlation is a small quantity $I_{\mathrm{Cl}}^{(l)}(\rho) = \frac{1}{2}\log d$. After the complete measurement M_A in the one-way message exchange, the final value of the amount of information in the quantum state is determined as $I_{\mathrm{Cl}}(\rho') = I_{\mathrm{Cl}}^{(l)}(\rho) = \log d$; i.e., the amount of available information increases.

Remark 6. Note that the complete measurement M_A in the basis $\{|k\rangle \otimes |t\rangle\}$ (Section 2) is optimal for the system A. The output value of the measurement result gives exactly information about what pure state of the ensemble is chosen. Therefore, there is an opportunity to apply the classical local process of processing (of the measurement result) for obtaining information about the distribution of results for other measurements. For the system A, the choice of the optimal measurement allows the system B to extract from $I_{\mathrm{Cl}}(\rho)$ amount of information I_{Acc} about the ensemble of uniformly distributed states $\{|k\rangle, (U_1 = H)|k\rangle\}_{k=0,\ldots,d-1}$.

4.3. Available Information about the Ensemble of Mixed States

In the general case, the available information about the ensemble of mixed states $\varepsilon = \{p_i, \eta_i\}$ is defined as the maximum mutual information between the measured state with the index i and the result of its measurement. The amount of available information $I_{Acc}(\varepsilon)$ can be characterized as the maximum value of information extracted from the quantum state with the help of positive operator-valued measurements (section 2) with elements of only rank 1 [8].

Assume that $M = \{\alpha_j|\phi_j\rangle\langle\phi_j|\}$ means a positive operator-valued measurement with elements of rank 1, where each state $|\phi_j\rangle$ is normalized and $\alpha_j > 0$. Then, $I_{Acc}(\varepsilon)$ can be calculated as

$$I_{Acc}(\varepsilon) = \max_M\Bigg[\underbrace{-\sum_i p_i \log p_i}_{\text{Classical component}} + \underbrace{\sum_i\sum_i p_i\alpha_j\langle\phi_j|\eta_i|\phi_j\rangle\log\frac{p_i\langle\phi_j|\eta_i|\phi_j\rangle}{\langle\phi_j|\mu_i|\phi_j\rangle}}_{\text{Quantum component}}\Bigg], \quad (4.3)$$

where $\mu = \sum_i p_i \eta_i$. Let us apply expression (4.3) to the solution of the abovementioned problem. The ensemble of states is given as $\left\{\frac{1}{2d}, U_t|k\rangle\right\}_{k,t}$ for the following values: $i = k, t$; $p_{k,t} = \frac{1}{2d}$, $\mu = \frac{I}{2}$ and $\langle\phi_j|\mu|\phi_j\rangle = \frac{1}{d}$.. Substituting all presented expressions in formula (4.3) for $I_{Acc}(\varepsilon)$, we obtain

$$I_{Cl}(\varepsilon) = \max_M \Bigg[\underbrace{\log 2d}_{\substack{\text{Classical}\\\text{component}}} + \underbrace{\sum_{j,k,t} \frac{\varepsilon_j}{2d} |\langle\phi_j|U_t|k\rangle|^2 \log \frac{|\langle\phi_j|U_t|k\rangle|^2}{2}}_{\text{Quantum component}}\Bigg]$$

$$= \max_M \Bigg[\underbrace{\log d}_{\substack{\text{Classical}\\\text{component}}} + \underbrace{\sum_j \frac{\alpha_j}{d}\left(\frac{1}{2}\sum_{k,t} |\langle\phi_j|U_t|k\rangle|^2 \log |\langle\phi_j|U_t|k\rangle|^2\right)}_{\text{Quantum component}}\Bigg],$$

where the following notation is used: $\sum_j \alpha_j = d$ and $t\sum_k |\langle\phi_j|U_t|k\rangle|^2 = 1$.

Since the relation $\sum_j \frac{\alpha_j}{d} = 1$ is satisfied, the second expression is a convex combination and can be bounded from above by maximization in the first term

$$I_{\mathrm{Cl}}(\varepsilon) \le \log d + \max_{|\phi\rangle} \frac{1}{2} \sum_{k,t} |\langle\phi|U_t|k\rangle|^2 \log |\langle\phi|U_t|k\rangle|^2.$$

Note that the term

$$-\sum_{k,t} |\langle\phi|U_t|k\rangle|^2 \log |\langle\phi|U_t|k\rangle|^2$$

is a sum of entropy measures of the measured state $|\phi\rangle$ in the computational and generalized bases. This entropy sum is bounded by the value $\log d$. Lower bounds of a similar type in the theory of quantum measurements are called entropy uncertainty *inequalities* (EUI), which determine quantitatively that it is impossible to extract the vector $|\phi\rangle$ from two generalized (joint) bases.

The presented relations imply that

$$I_{\mathrm{Cl}}(\rho) \le \frac{1}{2}\log d.$$

The equality can be reached if B is measured in the computational basis

$$I_{\mathrm{Cl}}(\rho) = \frac{1}{2}\log d, \quad I_{\mathrm{Cl}}^{(l)}(\rho) - I_{\mathrm{Cl}}(\rho) = 1 + \frac{1}{2}\log d.$$

Note that the property of proportionality increase is satisfied for multiple copies of the state ρ. Wootters showed in [8] that the available information from m independent copies of the ensemble ε of separated states is additive, $I_{Acc}(\varepsilon^{\otimes m}) = mI_{Acc}(\varepsilon)$. This implies that for the considered case we have $I_{\mathrm{Cl}}(\rho^{\otimes m}) = mI_{\mathrm{Cl}}(\rho)$.

Example 13. Let a two-particle state ρ_{AB} be given. We define a possible measure of classical correlation between subsystems A and B as

$$Cor_B(\rho_{AB}) = \max_B \left\{ S(\rho_A) - \sum_i p_i S(\rho_A^i) \right\}, \qquad (4.4)$$

where $\rho_A = Tr_B(\rho_{AB})$ is the reduced density matrix. The von Neumann entropy is $S(\rho) = -Tr(\rho \log \rho$. The conditional density matrix ρ_A^i s defined in terms of the density matrix of the state A after the realization of the measurement B_i over the state B in the form

$$\rho_A^i = \frac{Tr_B(B_i\rho_{AB})}{Tr_{AB}(B_i\rho_{AB})}.$$

The probability to determine A in the state ρ_A^i is $p_i = Tr_{AB}(B_i\rho_{AB})$. Correlation measure (4.4) has a simple physical interpretation: if A and B are not correlated, then the marginal value of the entropy $[S(\rho_A)]$ of the state A and the weighted averaged value of the entropy of A after the positive operator-valued measurement $[\sum_i p_i S(\rho_A^i)]$ over the state B yields as a consequence $Cor_B(\rho_{AB}) = 0$ since for the non-correlated system AB, the state of A does not depend on the action of the positive operator-valued measurement over the state of B.

In addition, we note that the relation

$$\rho_A = \sum_i p_i \rho_A^i;$$

takes place. Then for the given positive operator-valued measurement over the system B, the expression $[S(\rho_A) - \sum_i p_i S(\rho_A^i)]$ gives the entropy deficit, whose definition was given in [49]. Therefore, the classical correlation $Cor_B(\rho_{AB})$ can be considered as the maximum averaged increase of the entropy of the system A when the state ρ_A^i (after completion of the measurement B_i over the system B) is compared specifically with the situation in which only mixed states ρ_A are known. The classical correlation establishes the measure of correlation power of two subsystems without specifying the priority of the subsystem used for extracting this correlation.

The amount of quantum mutual information $I(A : B)$ of the two-particle compound system AB can be decomposed into the information deficit (or work) Δ and the classical information deficit Δ_{cl} in the form $I = \Delta_{cl} + \Delta$. This results in a natural analogy with the decomposition process for the case of amount of mutual information employed in describing various classical correlation measures [46]. The amount of estimated classical correlation and relative entropy (E_{REn}) in entangled states also does not exceed in total the amount of the von Neumann mutual information between the two subsystems, i.e.,

$$I(\rho_{A:B}) > [C_B(\rho_{AB})]_{opt} + E_{\text{REn}}.$$

Physically, this means that the choice of a non-optimal positive operator-valued measurement may destroy the total correlation. It is natural that in the consideration of all possible positive operator-valued measurements for a given state, the optimal class of positive operator-valued measurements cannot eliminate the total correlation. The possibility of different definitions of measures of quantum correlations different from the representation of them in the form (E_{REn}) is another alternative, which makes the concepts of classical correlation and mutual information consistent. One of the possible candidates is quantum chaos [50].

Example 14. Mutual information and quantum chaos. In contrast to the definition of classical conditional entropy, the quantum conditional entropy is a dependent quantity on the measurement procedure conducted over the investigated system. As the measure of quantum chaos, we take [50, 51]

$$\delta(A:B) = I(A:B) - J(A:B)_{\{\Pi_i^B\}}.$$

The quantity J specifies information about the system B based on the results of a series of measurements $\{\Pi_i^B\}$

$$J(A:B)_{\{\Pi_i^B\}} = S(A) - S(A|\{\Pi_i^B\})$$
$$= S(A) - \sum {}^*p_i S({}^*\rho_i^A),$$

where ${}^*p_i$ and ${}^*\rho_i^A$ are chosen in advance as special cases of p_i and ρ_i^A, when the set of measurements are formed as bounded one-dimensional projections Π_i^B.

The presented definition of quantum chaos clearly describes the inseparable dependence of quantum conditional entropy on the measurement procedure. The measure of quantum chaos is zero if there exists a measurement (at least one), for which this measure takes zero value. Therefore, quantum correlation is attached to the minimum value of the measure of quantum chaos.

When choosing the corresponding set of measurements over the system B as a set of one-dimensional projections, it is easy to test that the set of measurements under which the quantum chaos is minimized, i.e., the quantity J is maximized, is equivalent to positive operator-valued measurements optimizing the measure of classical correlations of binary states. This result follows from these quantitative measures and the result of optimization of classical correlations under the application of only projective measurements. Therefore, we have

$$\max[J(A:B)_{\{\Pi_i^B\}}] = Cor_B(\rho_{AB}).$$

Then, we have

$$I(A:B) = Cor_B + \min_{\{\Pi_i^B\}} \delta(A:B),$$

i.e., for binary states, the classical correlation and quantum chaos are the constituent components in the estimation of mutual information. Therefore, there is no measurement that extracts classical correlation that is able to change the value of mutual information between two subsystems when the classical correlation is added to the relative entropy. This is confirmed by the statement that it is more convenient to use the concept of quantum chaos as the quantum component for the classical correlation instead of relative entropy.

5. A MODEL OF QUANTUM FUZZY INFERENCE BASED ON TECHNIQUES OF QUANTUM AND SOFT COMPUTING

According to the general ideas of implementation of models of quantum computing on a quantum algorithmic gate [8, 10, 23], the logical combination of particular knowledge bases in a single generalized space can be implemented with the help of the superposition operator; then, using the quantum correlation operator (which, in the search quantum algorithm, is described by various models of a quantum oracle [10]) the search of a ìsuccessfulî marked solution is formalized, and, using the interference operator together with classical measurement operations, the desired "appropriate" solutions are extracted from the processes of quantum computing (as a result of operation of the quantum algorithm). The additional operator in the local feedback of the communication block of quantum fuzzy inference (Fig. 1) plays the role of reinforcement of quantum correlations in the course of controlling the search for successful solutions.

The method of designing robust knowledge bases of self-organizing structures of a fuzzy controller described in what follows is based on a special (simplified) variant of the general model of quantum fuzzy inference. Then, the considered model (as a particular case of the quantum block in Fig. 1) uses as the input signal a finite set of partial knowledge bases designed using the knowledge base optimizer) [1, 2, 7]. The structure and the Quantum algorithm of the simplified model of quantum fuzzy inference are applied in the system for simulating knowledge bases of self-organizing structures of the fuzzy controller. The self-organizing fuzzy controllers based on quantum fuzzy inference are involved in control loops of robust on-line intelligent control systems (see in what follows the description in Section 7).

Related works and results: quantum genetic algorithms and quantum evolutionary programming. Quantum evolutionary programming, which, in turn, can be divided conditionally into two fields [4, 10, 52], genetic algorithms generated by an analogue of quantum processes and quantum genetic algorithms, is a generalization of the so-called classical evolutionary algorithms in the theory of computational intelligence.

In the first case, chromosomes in quantum cells are represented in the form of quantum bits, and the solution is found by the method of designing quantum algorithmic gates (Fig. 2). In the second case, an attempt is made to solve the key problem in this field consisting of how a genetic algorithm can be implemented in the tools of the hardware support of quantum computing. One of the main points for developing a family of quantum genetic algorithms is to construct a quantum algorithm that provides the advance of parallel computing of both genetic algorithms and quantum computing, and due to the presence of quantum effects in computational processes, also provides an efficient random search [10, 53].

Various models of quantum evolutionary programming, such as the optimization algorithm based on the quantum swarm particle method, intelligent genetic operators, etc, have been developed (see [4, 10, 52]). Some of these models were descried briefly in [4, 52]. In [10], the distinctions and general properties, such as parallelism of computations in genetic algorithms and in the quantum algorithm, estimates of computational complexity in quantum genetic algorithms, etc., were described. It is worth noting that the model of a quantum genetic algorithm presented in Fig. 1 generalizes the models of quantum evolutionary programming listed above.

The main problem of the model of quantum fuzzy inference presented in the paper is to provide the possibility of on-line realization of self-organization processes in designing control processes on the set of new unpredictable control situations or under conditions of incomplete information about the external environment, using a finite set of knowledge bases designed based on the knowledge base optimizer [3–5, 7]. The output of the quantum fuzzy inference block is the robust control signal of the coefficient gains schedule of a fuzzy PID controller for the current unpredicted control situation.

5.1. A Method for Solving the Problem

For solving the problem posed, we apply the process of extracting new hidden quantum information from a finite set of knowledge bases (designed with the help of the knowledge base optimizer) based on the superposition operator, as well as the method of reduction and compression of redundant classical information in knowledge bases of individual fuzzy controllers also designed based on the knowledge base optimizer (see the Introduction). The processes of compression and reduction of redundant information in classical individual Knowledge bases forming the laws for controlling the coefficient gains schedule of a fuzzy PID controller are based on the laws and results of quantum information theory [8, 10, 36, 37]. In particular, an efficient algorithm of quantum information theory is applied for data compression. Using the criterion of minimum of the von Neumann entropy, this algorithm maps the set of quantum (non-informative) bits into a small subset of informative quantum bits that have a higher probability to appear in the message. The reduction of redundant classical information in the laws for control of knowledge bases of a fuzzy controller results in an increase in the value information and, as a consequence, in an increase in the robust level of control processes without loss of quality, such as the reliability of the accuracy support for control processes (see Figs. 1 and 3 in [7]). As a result, the designed robust knowledge base for a fuzzy controller (with the help of the model of quantum fuzzy inference) can adapt the laws for controlling the coefficient gains schedule of a fuzzy PID controller to unforeseen changes in the external environment of functioning the control object and to uncertainty in the source information about the control situation. In this paper, we consider the main ideas of quantum computing and quantum information theory as applied to the development of methods for designing a robust knowledge base based on quantum fuzzy inference.

Applied aspect. Based on the result of simulating the laws for controlling the gains of fuzzy PID controllers obtained by using the model of quantum fuzzy inference, a new principle for designing robust structures of intelligent control systems is formulated: *the design of a fuzzy controller for efficient control of complex control objects that is simple in structure and practical implementation with an improved wise level.*

In this section, we consider briefly the necessary properties and laws of quantum computing and quantum information theory employed in the model of the quantum block (Fig. 1) in the form of quantum fuzzy inference for the software support of the self-organization properties of robust fuzzy controllers (see Section 6 in what follows). A simplified model of quantum fuzzy inference and the operation algorithm are investigated that allow one to obtain practical results in the processes of designing robust knowledge bases in various structures of intelligent control systems.

5.2. Basic Structures and Models of Quantum Computing Employed in the Model of Quantum Fuzzy Inference

The model of quantum fuzzy inference harnesses and implements the processes of extraction of hidden quantum information contained in classical individual knowledge bases for fuzzy controllers based on the physical laws of quantum information theory and quantum computing. Classical knowledge bases for fuzzy controllers were designed in [1, 7] in advance for particular fixed control situations drawing the learning processes based on the knowledge base optimizer, and fuzzy controllers form optimal signals for controlling the coefficient gains of conventional PID controllers. To extract and aggregate the additional hidden valuable information in knowledge bases based on quantum fuzzy inference, it is necessary to enable new additional logical operators for data processing lacking in the tools of soft computing technology [3–5].

Additional (necessary for processing classical information) unitary invertible (quantum) operators have the following names: superposition, quantum correlation (entangled operators), interference, being mathematical tools of quantum computing [8–10]. In quantum computing theory (Section 2), we can distinguish the following two lines of investigations:

given a set of points of a functional $S = \{(x, y)\}$, find the form of an operator U such that the condition $y = U \cdot x$ holds;

given a problem (quantum algorithm), find the form of the quantum scheme, a quantum algorithmic gate that solves this problem (implementing the given quantum algorithm).

Algorithms for solving these problems can be implemented based both on hardware and with the help of the corresponding software product with implementation on a conventional computer [10]. In [10, 23], the possibility of efficient simulation of the quantum algorithm on a conventional computer, which is used in this paper for simulating the quantum algorithm in quantum fuzzy inference was shown.

The fundamental result of the quantum computing theory consists in the fact that all operations (similar to the classical case) can be implemented in a circuit consisting of universal basis elements. In contrast to the classical analogue, the quantum algorithmic gates can be executed on various classes of universal elements depending on the employed computational basis. Quantum algorithmic gates (with fixed computational and measurement bases) provide a description of the evolution of a unitary operator U such that a quantum computational process $|\psi_{\text{fin}}\rangle = U|\psi_{\text{in}}\rangle$ corresponds to it, where the vector (wave function) $|\psi_{\text{in}}\rangle$ specifies the initial conditions for computing (the solved problem), and $|\psi_{\text{fin}}\rangle$ reflects the result of computing by the action of the operator U on the initial state $|\psi_{\text{in}}\rangle$.

Choosing different forms of the operator U (in particular, Hamiltonian), we can form different models of quantum computing. In the general form, the model of quantum computing [8, 10] comprise the following five stages:

preparation of the initial (classical or quantum) state $|\psi_{\text{in}}\rangle$;

execution of the Hadamard transform for the initial state in order to prepare the superposition state;

application of the entangled operator or the quantum correlation operator (quantum oracle) to the superposition state;

application of the interference operator;

application of the measurement operator to the result of quantum computing $|\psi_{\text{fin}}\rangle$.

Quantum operators work in an iterative mode, depending on the type of quantum algorithm. Note that it is supposed for the general case that certain computational problems can be solved on a quantum computer more efficiently (with smaller computational complexity, the so-called *NP*-problem) than on a classical computer. Moreover, with the help of efficient application of a quantum computer, we can obtain solutions of problems that cannot be solved at the classical level; i.e., the problem for which there is no classical (randomized) algorithm can be efficiently solved by applying the quantum algorithm.

These observations testify that quantum algorithms provide a physically substantiated background for not only a technique for computation acceleration, but also for finding solutions of complex problems using quantum laws, such as *superposition* for extending the space of possible solutions, *quantum parallelism* of computational processes in order to accelerate the search for solutions, and *quantum interference* in order to extract the desired solution. Additionally to the mentioned computational resources, *quantum correlation* is considered as a new physical computational resource, which allows one to increase sharply the successful search of solutions to the problems that have not been addressed in the classical computational field. Among these problems are the following: teleportation, superdense coding, data transmission via quantum communication channels with an improved confidentiality and security level (against unauthorized access and eavesdrop), and the correction of quantum codes with a given tolerance level, etc. [8–10].

From the mathematical point of view, joint quantum states are formed with the help of tensor (Kronecker) product on Hilbert spaces of basis states. The result of this operation is a quantum register. The involvement of quantum correlation in the computational process leads to an increased rate and reliability of the search for solutions based on the corresponding quantum algorithm, and due to the given physical computational resource, many computational operations can be executed in parallel. Therefore, in this sense, quantum correlation demonstrates a new special physical resource of quantum computing.

From the point of view of functional capabilities, quantum algorithms can be classified into the following two groups: decision-making algorithms and search algorithms. In quantum computations, first of all, the qualitative properties of functions, which are coded in the initial quantum states, are of interest. To find a solution by using quantum algorithms, the initial superposition of initial states is purposefully changed, applying successfully the listed types of quantum operators. In this case, we can use the algebraic formalism, which is supported by abstracting logical inference relative to quantum effects and maps the most important quantum effect at the programming level, eliminating the difficulty of a hardware implementation, such as decoherence.

Quantum algebra allows one to formalize certain important properties of quantum effects by including their description in certain program attributes (Section 2). Then the software tools contain a technique for describing program attributes and efficient logical inference on a large number of quantum bits and basis states with a high level of quantum correlation, as well as possesses a descriptive representation for physical properties of the described quantum operators [10, 27–34]. This approach is hardware-independent and can be used as a model of quantum computing or the basis of a quantum programming language.

In Section 2, examples illustrating the main quantum operators and the properties were considered. Staged application of quantum algorithmic gates and the measurement of computational results after application of quantum operators in quantum algorithmic gates allow one to implement quantum fuzzy inference on a classical computer (see Fig. 1). The quantum block repeats iteratively k times quantum operations in order to reproduce a set of k basis vectors containing the desired solution. Since the measurement is not a deterministic operation, the basis vectors obtained in the set are not identical and each of them codes a part of information necessary for solving the investigated problem. The last component of the quantum algorithm contains the block for interpretation of the set of basis vectors, which allows one to select the final informal solution of the investigated problem with a probability.

As was mentioned above, the fundamental result of quantum computing theory consists in the established opportunity to embed all quantum algorithms into quantum gates implemented using universal typical elementary circuits (of type "AND", "OR", "NOT", "controlled NOT", etc.), which are employed in the structure of a conventional computer. These cells are described mathematically by unitary operators reflecting the evolution of the computational process. In quantum computing theory, it was also shown that it is possible to implement quantum computing based on a classically efficient emulator.

Thus, the method for designing quantum algorithmic gates developed in [10, 23] can be applied for simulating the processes of global optimization of knowledge bases in robust structures of intelligent control systems using the technique of quantum computing, which contains software tools, such as quantum genetic search algorithm and quantum learning optimization processes. The model of quantum fuzzy inference is a basis of software optimization toolkit of this type.

Remark 7. The problem of evaluating quantum effects in the quantum algorithm and developing the models of quantum algorithm themselves is referred to the class of problems of increased complexity. In quantum computing, there are many technical difficulties connected with the necessity to manipulate with nontraditional properties of quantum information. Among them is the impossibility of copying quantum information about an unknown quantum state, destruction of valuable information contained in the superposition in the measurement of computational results, and operations with concepts that are nonstandard for classical computing theory, such as quantum correlation, relative phase, or superposition. Superposition and quantum correlation (entangled states) do not have classical analogues and determine the power of quantum computing. The phase has a conventional interpretation of a continuous quantity, but in quantum computing, the base phase unit plays an additional role of distinguishing quantum states of intermediate states without possibility of copying. Note that the descriptive representation of quantum operators points to the necessity to include the desired qualitative properties of functions in the process of preparation of the initial superposition of initial quantum states as potential solutions.

Many of the most popular models of quantum computing are direct quantum generalizations of the corresponding constructions of classical computations. Among them are the quantum Turing machine, quantum gates, and random walk. These models are based on unitary evolution (as a basis mechanism of information processes). Only at the end of computations, finite measurements are conducted, which results in a mapping of quantum information to classical one (for reading the computational result in the classical form). The two main ideas are considered. The first idea is connected with amplification of the probability amplitudes of the desired solution, and the second idea states that classical computations can be simulated on a quantum computer. Thus, instead of describing the quantum algorithm, we can deal with the corresponding classical algorithm, which gives a solution with a given error probability. Then, the classical algorithm is transformed into a quantum algorithm, and the procedure for amplification of the probability amplitude of the desired solution in the quantum algorithm is applied.

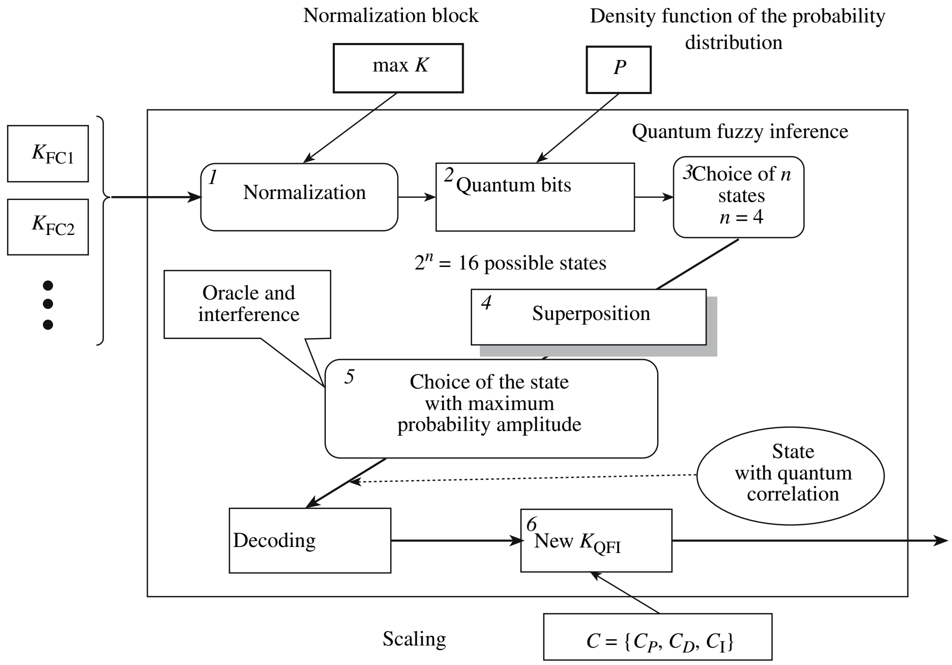


Fig. 2. The functional structure of QFI in real time.

Other models of quantum computing are based on the application of only irreversible measurements (one-way quantum computing), on the theory of hidden variables, and adiabatic quantum computing algorithms. There is also a model of quantum computing based on a dual quantum computer, using corpuscular and interference properties of quantum systems, etc. These models were investigated in [10] from the point of view of the quantum theory of optimal control processes. With account of remarks made, we analyze briefly the functional model of quantum fuzzy inference based on typical quantum operators of quantum computing.

5.3. A Model of Quantum Fuzzy Inference

As was mentioned above, the main problem solved by quantum fuzzy inference is to form a knowledge base with improved robustness level from a finite set ofkKnowledge bases for fuzzy controllers generated with application of techniques of soft computing and stochastic simulation. Consider briefly the functional structure and operation of the basic blocks of quantum fuzzy inference. As an example without loss of generality of the result, we discuss the process of extracting hidden quantum information, data processing, and generation of robust knowledge bases of fuzzy controllers using knowledge bases of two fuzzy controllers designed for fixed control situations (different each from another).

Figure 2 shows the functional structure of a simplified model of quantum fuzzy inference as a particular case of the generalized model presented in Fig. 1. In real time, control signals from the knowledge base of fuzzy controller (“Initial state” step in Fig. 2) generated in advance are input to quantum fuzzy inference. The next step is a normalization process (block *1* in Fig. 2) for signals received in the interval [0, 1] by division of the amplitudes of the trajectories of control signals by the maximum amplitude (block “max K” together with block *1* in Fig. 2).

After normalization of signals, quantum bits are formed (block *2* in Fig. 2) from the current values of the normalized control signals. For this purpose, the density functions of the probability distribution are determined in advance based on representative sample trajectories of control signals. Then, by integrating the obtained density functions of the probability distribution integral functions of the probability distribution are determined (blocks “P” and *2* in Fig. 2). The functions of the probability distribution obtained in this way allow one to select ìvirtualî states $|1\rangle$ of control signals for forming the superposition using the Hadamard transform from the current state of selected control signals. Note that we use a probability law of the form $P(|0\rangle) + P(|1\rangle) = 1$, where $P(|0\rangle)$ and $P(|1\rangle)$ are the probabilities of the current real and virtual states of the control signals, respectively. For the current real normalized state of the control signal $|0\rangle$, using the integral function of the probability distribution, we determine its probability. Then, from the law of probability conservation, the probability of the virtual state of the normalized control signal is calculated. Using the same integral law of probability distribution, by the inverse mapping, we calculate the value of the corresponding virtual state of the control signal. Therefore, the super-

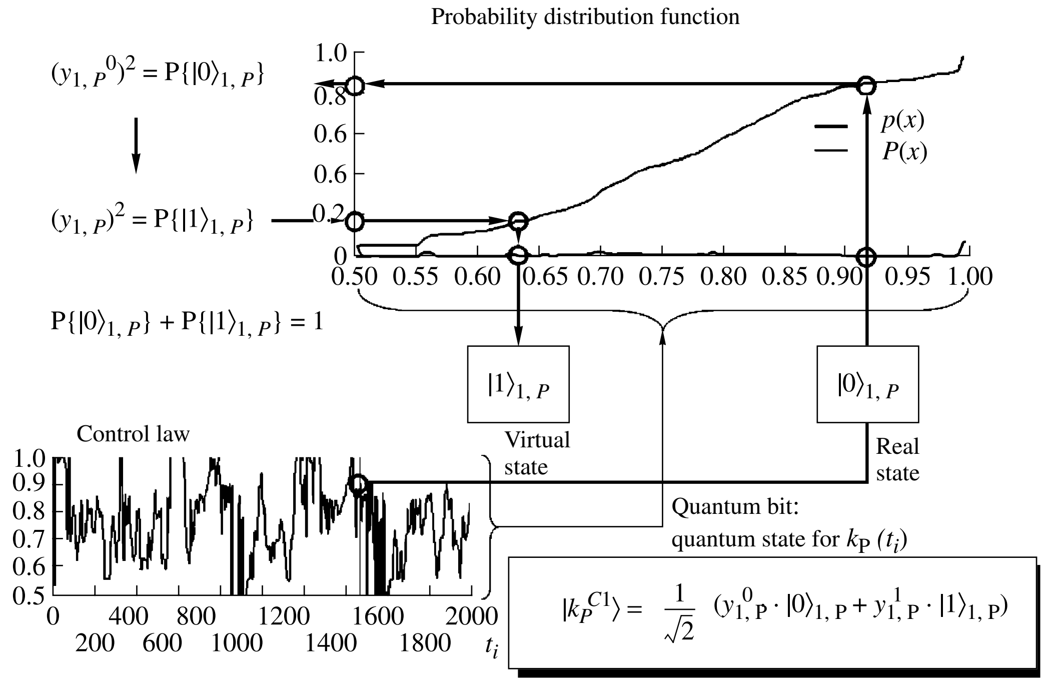


Fig. 3. The process of forming quantum bits (block 2).

position of the quantum system "real state–virtual state" has the following form

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left(\sqrt{P(|0\rangle)}|0\rangle + \sqrt{1-P(|0\rangle)}|1\rangle\right) = \textit{Quantum bit}.$$

Figure 3 presents the block diagram of the computational process and formation of the quantum bit for the current state of the normalized control system describing the gains of the fuzzy PID controller in the structure of the intelligent control system (Table). Next, the type of quantum correlation (Fig. 2, block *3*) and the corresponding components of the applied type of correlation of the normalized control signals are chosen. For the considered situation, three types of quantum correlation are possible (containing valuable quantum information hidden in the designed knowledge bases): *spatial, temporal, and spatial–temporal.*

Figure 4 presents the three listed types of correlation between the processes of controlling the gains of two fuzzy PID controllers. The fundamental distinctive feature, characterizing the relation between the classical and quantum correlation types, is the following fact. In the quantum variant, there is no *cross* (mixed) correlation between the *real* and *virtual* states of the normalized control signal. Classical correlation in this case manifests itself as a particular case of total quantum correlation. Therefore, the total correlation consists of the following components: classical (between real values of the normalized control signal); quantum (between the virtual values of the normalized control signal); and mixed (between the real and virtual values of the normalized control signal).

The first two correlation types are investigated in the correlation theory of stochastic (classical and quantum) processes. The third type is new in the theory of quantum stochastic processes and reflects the effect of interference of the classical and quantum correlation. This type of total correlation contains hidden classical correlation in the formed superposition of quantum bits and provides information resource for extracting additional (unobservable) valuable quantum information (see Sections 3 and 4).

Figure 5 presents an example of forming the spatial correlation in quantum fuzzy inference from signals for controlling the gains of two fuzzy PID controllers with a knowledge base designed using the tools of the knowledge base optimizer for two different control situations. According to Fig. 5, the quantum spatial correlation (as a type) is classified in turn into two correlation types: internal and external. The internal correlation is formed by establishing static interrelations between the output signals for controlling the gains of the first given fuzzy controller of type K_{FC1}. The external correlation type characterizes the relation between the corresponding control signals of the first K_{FC1} and second K_{FC2} fuzzy controllers, where the subscript "FCl" means fuzzy controller 1 (FC1), and the subscript "FC2" means fuzzy controller 2 (FC2). In what follows, we present a more detailed description of the notation in Fig. 5.

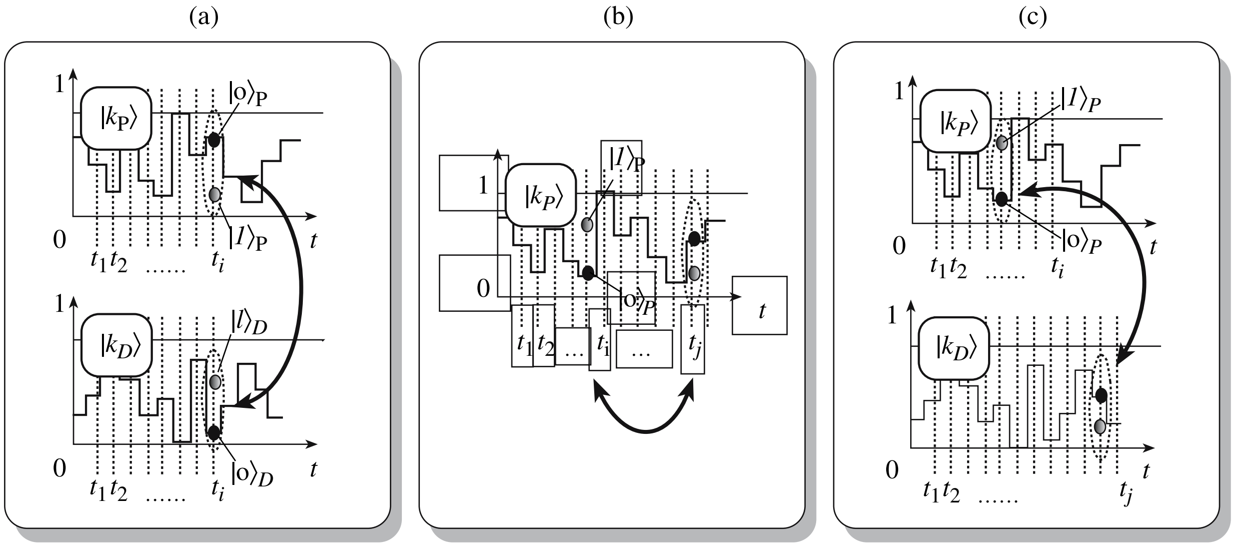


Fig. 4. Types of quantum correlations in the laws for controlling the coefficient gains of a PID controller in on-line: (a) spatial, (b) temporal, (c) spatial-temporal.

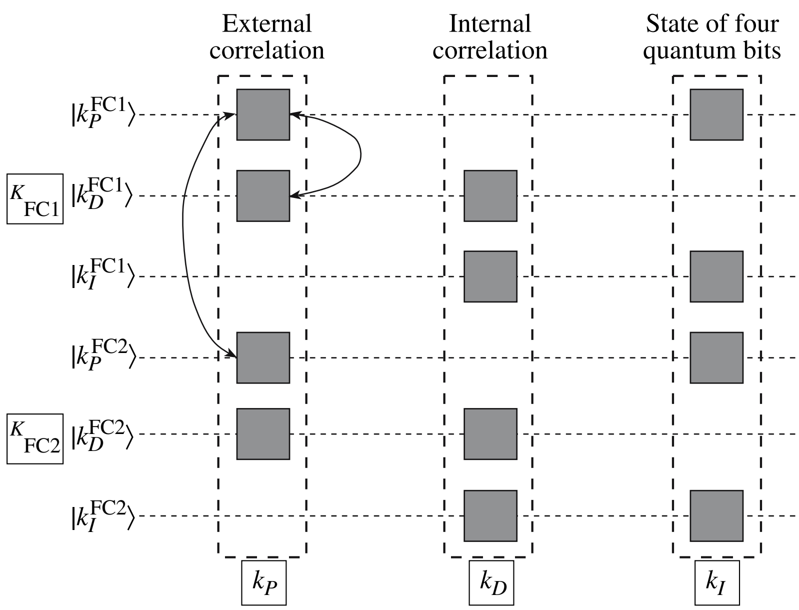


Fig. 5. The process of forming the internal and external spatial correlation for new gains k_P, k_D, and k_I.

Note that analogous considerations are valid for the temporal and spatial–temporal correlations. As a result of forming these types and forms of quantum correlation the *coordination control* between the gains is performed using the corresponding internal and external correlations. Therefore, for the particular case of two fuzzy controllers considered, each quantum superposition state of the chosen gain is described by four quantum bits.

Consider the algorithm of forming the superposition (Fig. 2, block *4*) for calculating the optimal value of, e.g., the proportional gain of a fuzzy PID controller (Fig. 2, block *5*) using the knowledge bases of two fuzzy controllers from different control situations. The other gains are obtained using a similar algorithm.

Figure 6 shows an example of calculating the desired set of values of the proportional gain from the set of combinations

$$\{k_P^{FC1}, k_P^{FC2}, k_D^{FC1}, k_D^{FC2}\}.$$

$$|k_P^{FC1}\rangle \otimes |k_P^{FC2}\rangle \otimes |k_D^{FC1}\rangle \otimes |k_D^{FC2}\rangle$$

$$= \frac{1}{\sqrt{2}}(y_{1,P}^0 \cdot |0\rangle_{1,P} + y_{1,P}^1 \cdot |1\rangle_{1,P}) \otimes \frac{1}{\sqrt{2}}(y_{2,P}^0 \cdot |0\rangle_{2,P} + y_{2,P}^1 \cdot |1\rangle_{2,P}) \otimes$$

$$\otimes \frac{1}{\sqrt{2}}(y_{1,D}^0 \cdot |0\rangle_{1,D} + y_{1,D}^1 \cdot |1\rangle_{1,D}) \otimes \frac{1}{\sqrt{2}}(y_{2,D}^0 \cdot |0\rangle_{2,D} + y_{2,D}^1 \cdot |1\rangle_{2,D})$$

$$= \frac{1}{\sqrt{2^n}}\Big(\underbrace{\boxed{\alpha_1} \cdot |0000\rangle + \alpha_2 \cdot |0001\rangle + \ldots + \alpha_{2^n-1} \cdot |1110\rangle + \alpha_{2^n} \cdot |1111\rangle}_{\text{16 possible states}}\Big) \qquad (n = 4)$$

$$\alpha_1 = y_{1,P}^0 y_{2,P}^0 y_{1,D}^0 y_{2,D}^0\text{: probability amplitude}$$

Fig. 6. An example of calculating the proportional gain k_P based on the set $\{k_P^{\mathrm{FC1}}, k_P^{\mathrm{FC2}}, k_D^{\mathrm{FC1}}, k_D^{\mathrm{FC2}}\}$, new k_P (the superposition of four chosen states).

Here, k_P^{FC1} is the applied value of the signals for controlling the proportional gain of the knowledge base of the first fuzzy controller designed for the first fixed control situation; k_D^{FC2} is the value of the signals for controlling the differential gain of knowledge base of the second fuzzy controller designed for the second control situation (which is essentially different from the first one in the external conditions of functioning of the control object). Applying tensor product between the Hadamard transforms, we obtain the terms of the form $k_P^{\mathrm{FC1}} \otimes k_D^{\mathrm{FC2}}$ and similar combinations of gains. As can be seen from the notation in Fig. 6, there are 16 possible states describing the combinations of correlations (with account of their type and form) between the corresponding gains of the two fuzzy controllers designed for different control situations.

It is worth noting that there is a fundamental specific feature of the process of forming and designing a new type of robust gains of a fuzzy PID controller by applying the quantum superposition operator. The superposition operation allows one to join different knowledge bases logically and to distinguish the correlation priority of particular states in the superposition of knowledge bases with application of various optimization criteria. When using the knowledge base optimizer based on soft computing, the formation of an individual knowledge base is performed by the random search in the genetic algorithm on a fixed search space. Note that the random search is performed independently for three gains in the fuzzy PID controller. This means that other sets of gains that have the equivalent total control effect may exist.

New types and forms of quantum correlation allow one to perform coordinated control of gains by using only the physical resources of the employed correlation form. This results in compression and elimination (reduction) of redundant information in the independent laws of controlling the gains, extraction of the most valuable information, and, as a consequence, in an increase in the robustness level of the new designed knowledge base due to the new type of coordination. The application of entangled states in the three types of correlation gives an opportunity to improve the robustness level of the designed knowledge base (with the help of physical properties of entangled states), using the teleportation effect between quantum states in the formed superposition (see Fig. 1). This approach to designing robust knowledge bases does not have classical analogues and differs by the pure quantum nature of the obtained design effects.

The choice of the priority quantum state in the superposition under a fixed correlation type (in this case, spatial) is executed in block *5* in Fig. 2. Consider a possible approach to the choice of an optimization criterion for extracting the priority state from the superposition formed in block *4* (Fig. 2) of coded possible states of the gains of a fuzzy PID controller. For this purpose, we use the concept of ìintelligent quantum stateî introduced in quantum measurement theory as a state with minimum uncertainty (in the sense of the minimum of the Heisenberg uncertainty inequality) [54, 55]. This concept is also associated with the solution of quantum wave equations (of the Schredinger type, etc.), for which the wave packet of the state of a quantum system is a coherent state. In this state, the uncertainty relation attains the global minimum. The definition and calculation of the state in the quantum algorithm was given in [25, 26] based on the definition of the von Neumann entropy and information Shannon entropy in this quantum state. By [10, 26], the ìintelligent quantum stateî in the quantum algorithm is the minimum of the difference between the information Shannon entropy of the quantum state and the physical von Neumann entropy of the quantum state

$$\mathbb{I}(|\text{Quantum state}\rangle) = \min(H^{Sh} - S^{vN}), \qquad (5.1)$$

where H^{Sh} and S^{vN} are the Shannon and von Neumann entropies, respectively. According to the laws of quantum information theory, we have the following inequality:

$$H^{Sh} \geq S^{vN} \text{ т.е., } \mathbb{I}(|\text{Quantum state}\rangle) \geq 0.$$

Remark 8. We recall that the squared probability amplitude of a state in quantum mechanics is equal to the classical probability of the event that the quantum system is located in a given state (the Bohr postulate, which have several variants of thorough substantiation [57]).

From the point of view of quantum information theory, it is well known that a pure quantum state is characterized by the zero value of the von Neumann entropy. Therefore, in the considered quantum algorithm, the ìintelligent stateî takes place for the minimum of the Shannon information entropy of the quantum state. In turn, the desired minimum is attained under the maximum of the probability state (by the definition of the information Shannon entropy of the quantum state $H^{Sh} = -\sum_i p_i \ln p_i$, , i.e., the global minimum is observed for the probability maximum p_i). Since p_i is the corresponding probability amplitude squared by the definition, the maximum principle for the probability amplitude in the correlated state can be taken as the criterion for the choice of the priority intelligent correlation (coherent) state in the superposition of possible candidates.

Thus, the calculation of amplitudes of quantum states in the superposition of states with mixed forms of quantum correlation (block *5* in Fig. 2) and the choice of the maximum one among them is realized by the model of a quantum oracle possessing (by the definition of a quantum oracle) necessary information about the desired solution. Figure 7 shows the block diagram of this algorithm.

Using the standard decoding procedure (inner product of vectors in a Hilbert space) and choosing the scaling coefficients for the output values of the designed gains (Fig. 2, block *6*), the iterative operation of the quantum algorithm of quantum fuzzy inference is performed. The presented model of the quantum algorithm for quantum fuzzy inference allows one to solve classical problems of designing knowledge bases of fuzzy controllers in the structures of intelligent control systems that have no analogues among the families of randomized classical algorithms and are characterized by polynomial computational complexity (BQP computational complexity class).

5.4. Quantum Information Resource in the Quantum Fuzzy Inference Algorithm

The algorithm for finding, extracting and coding additional valuable information from two fuzzy PID controllers (designed with the help of the knowledge base optimizer) based on the quantum fuzzy inference algorithm is described briefly in the Appendix and in detail in [3–5, 10]. The basic stages and the structure of the quantum fuzzy inference algorithm were presented in Subsection 5.3. Here, we give the structure of the information resource of the quantum fuzzy inference algorithm from the point of view of quantum information theory [8, 10, 45] (Section 4).

As was shown in Section 3, the process of optimal extraction of valuable information from the set of knowledge bases obtained based on the soft computing technique is based on the following four facts in quantum information theory [4, 10]: (1) there is an efficient algorithm of quantum data compression; (2) there exists a coupled representation of quantum and classical information in the quantum state; (3) total correla-

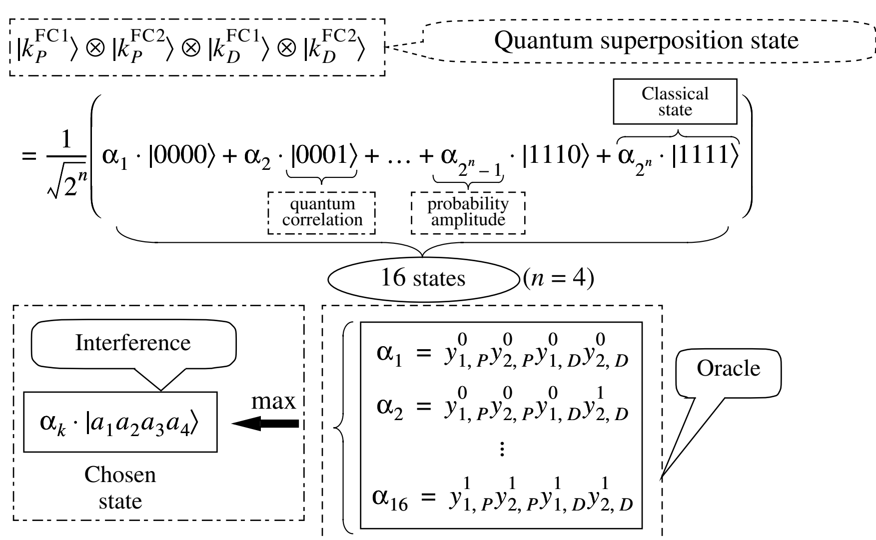


Fig. 7. The choice of quantum state according to the principle of maximum of probability amplitude (minimum of Shannon information entropy).

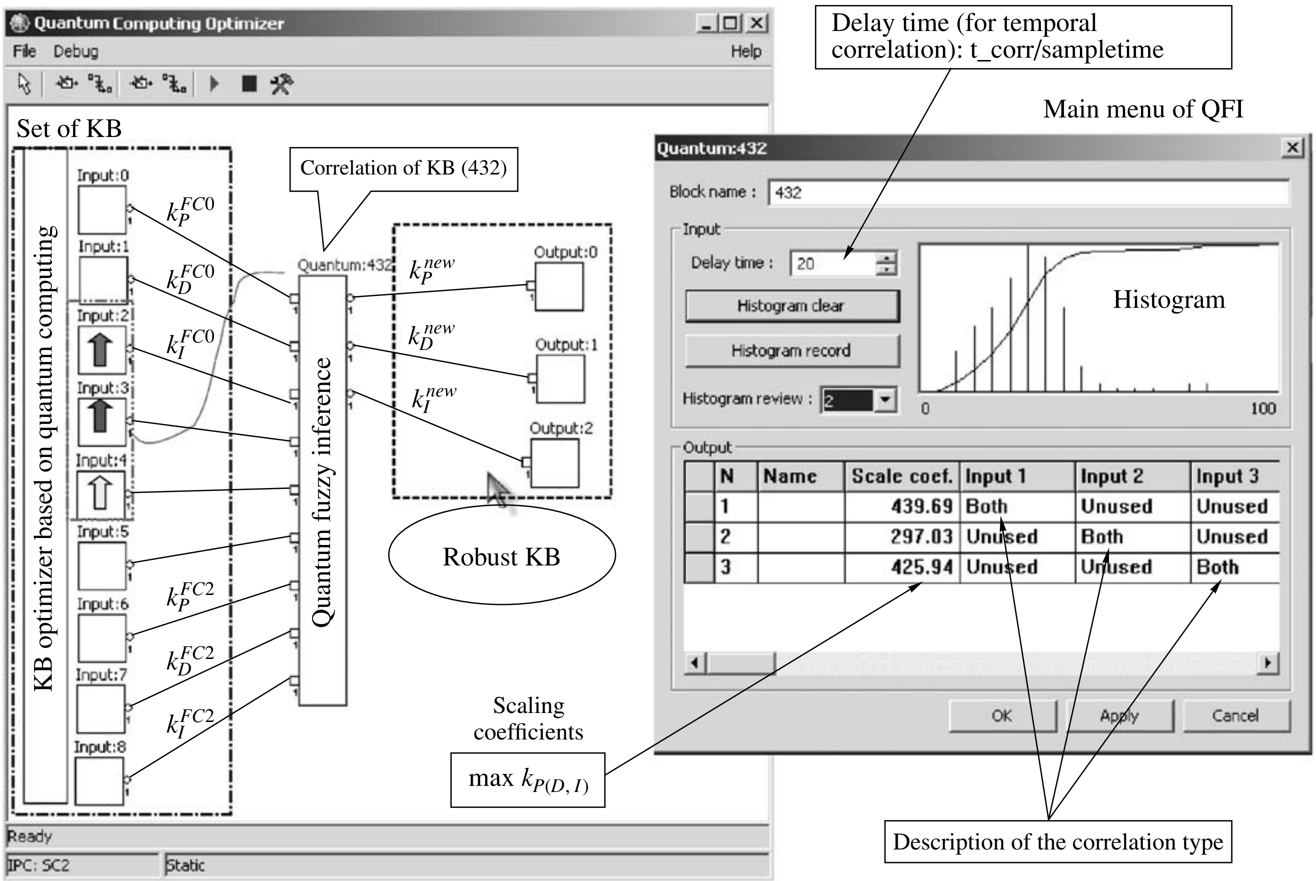


Fig. 8. The main menu of QFI with the use of quantum optimizer of KB.

tion in the quantum state is a mix of classical and quantum correlations; and (4) there is hidden (observable, i.e., accessible for extraction) classical correlation in the quantum state [40–48].

The quantum fuzzy inference algorithm employs the listed four facts as follows: classical information is compressed by coding in the computational basis $\{|0\rangle, |1\rangle\}$, and the quantum correlation between different computational bases of the designed knowledge bases is formed; the total information and correlation are separated and coupled into classical and quantum components using the Hadamard transform (facts 2 and 3); and the extraction of the hidden information in the quantum state is provided, and the operation of reduction of redundant information in the classical control signal is performed (fact 4), applying the criterion of maximum of the corresponding probability amplitude. Therefore, the listed facts provide a background of the information resource of the quantum fuzzy inference algorithm and give the opportunity to extract additional amount of valuable quantum information from individual robust knowledge bases, as well as to use it for designing robust control processes with an improved intelligent level. The ground of the applied effect is given by the procedures of compression and reduction of redundant information in classical control signals. In the next sections of the paper, we consider the application of this information resource to the structure of the quantum fuzzy inference algorithm.

6. SOFTWARE TOOLKIT AND SPECIFIC FEATURES OF THE DESIGN OF ROBUST KNOWLEDGE BASES FOR SELF-ORGANIZING FUZZY CONTROLLERS WITH APPLICATION OF QUANTUM FUZZY INFERENCE

In subsection 5.3, we considered a particular quantum algorithm for quantum fuzzy inference, which can be implemented on a conventional computer in the form of quantum algorithmic gates in accordance with the design technology developed in [23]. In this section, the structure of the quantum algorithmic gates software product developed for implementation of quantum fuzzy inference on a conventional computer is presented. Figure 8 shows that main program menu of quantum fuzzy inference, in which all blocks of quantum fuzzy inference and quantum algorithm are presented by program windows reflecting clearly the operation of quantum fuzzy inference in a graphical form. Figure 9 presents an example of the structure, the main

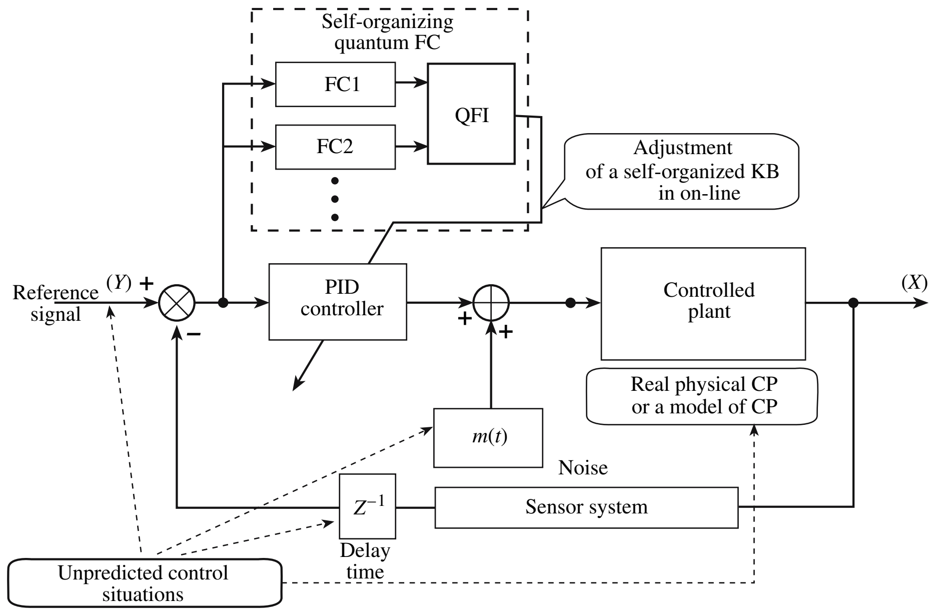


Fig. 9. Principle structure of a self-organizing ICS in unpredicted control situations.

stages for forming and designing a robust knowledge base of fuzzy controller from two knowledge bases of fuzzy controllers obtained by the tools of the knowledge base optimizer. This figure also specifies possible sources of unpredicted control situations. Figure 19 shows the block diagram of the simulation of the quantum algorithm in quantum fuzzy inference implemented in Simulink. In this section, we consider the operation of the quantum algorithm stage-by-stage and give a detailed description of the specific features of blocks of quantum fuzzy inference (Fig. 10) presented in the form of a block diagram in Figs. 2 and 9.

6.1. Input Signals in Quantum Fuzzy Inference

The design of the set of source knowledge bases for fuzzy controllers based on the knowledge base optimizer with the use of the soft computing technique is the first stage of operation of the mentioned tools (see the “QFI” window in the menu of Fig. 8). At the structural level [1, 7], the knowledge base optimizer is regarded as a new flexible software product for implementing computational intelligence, which is used efficiently in designing optimal structures and the required robustness level of knowledge bases in fuzzy controllers. In the structure of the knowledge base optimizer, a chain of logically interrelated genetic algorithms with fitness functions in the form of information–thermodynamic cost functions of control for rough optimization of the structure of knowledge bases is given. A standard learning algorithm based on the error backpropagation method is used for fine adjustment of the parameters of membership functions in knowledge bases.

It is also worth recalling that the correct definition of an optimal form of fuzzy inference, the type and quantity of production rules used and the membership functions of linguistic variables in knowledge bases and data bases for the employed fuzzy controllers is a central and most complex problem in the design of robust knowledge bases. It is this problem that is solved optimally at the first stage of the design technology by the knowledge base optimizer for a fixed control situation. A signal of physical measurements of the dynamical behavior of the controlled object or the results of simulation data (called the teaching signal (TS)) can be the input for the knowledge base optimizer. To design the (teaching signal) (or an estimate of the fitness function of the genetic algorithm), we apply a stochastic modeling system in order to extract knowledge from the dynamic behavior of the controlled object based on the corresponding physical or mathematical model. A detailed description of the structure and functional features of the knowledge base optimizer was presented in [1, 2]. The stochastic simulation system based of the method of nonlinear forming filters and a solution of the Fokker–Planck–Kolmogorov equation was additionally described in [7].

Thus, robust individual knowledge bases were designed for a fixed control situation. The output signals from the designed knowledge bases are input in on-line in the form of signals for controlling the coefficient schedule of the PID controllers employed to the quantum fuzzy inference block as the current reaction to a new control error. In this case, another information about the physical structure of the employed fuzzy controller and its knowledge base, as well as what production rules have fired in the unpredicted control situa-

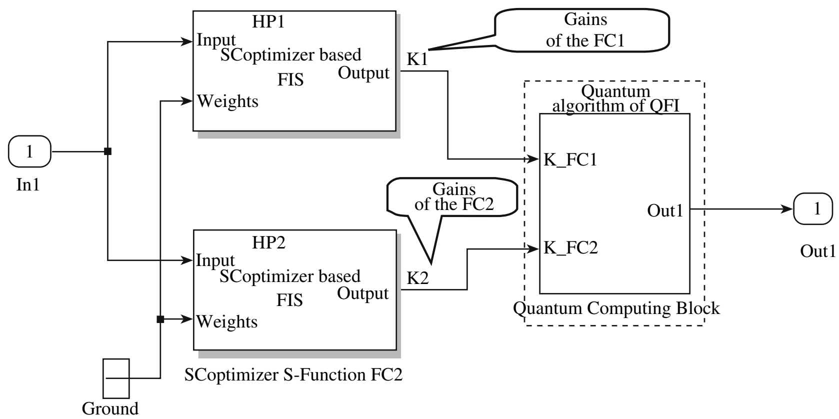


Fig. 10. The block diagram of the simulation of the quantum algorithm of QFI in Simulink.

tion, are not required for the operation of the quantum algorithm in quantum fuzzy inference.

6.2. Basic Operators in Quantum Fuzzy Inference (by the Example of Superposition of the Input Signals)

As was shown in Subsection 5.2 the quantum algorithm in the quantum fuzzy inference block (Fig. 1) is based structurally on the main quantum operators of quantum algorithm theory: *superposition* of classical states formed at the output of the knowledge base optimizer; the operator for forming *entangled* states (quantum oracle), *interference*, as well as classical irreversible measurements (see Fig. 7).

Remark 9. Let us recall that the employed knowledge bases of two fuzzy controllers were designed in an off-line mode for a given goal and different control situations (under various noise acting on the controlled object and the presence of a time delay in the channel for measuring the control error). Using the mentioned four facts from quantum information theory, quantum fuzzy inference extracts valuable hidden information from knowledge bases of the two designed fuzzy controllers. From the point of view of quantum information theory, in this case, for the sake of quantum correlation, the data exchange between the knowledge bases of the two fuzzy controllers is obtained in the communication channel formed using different forms of quantum correlation (Sections 3 and 4). The noted effect of data exchange is a pure *quantum effect* and is impossible at the classical level of description of design processes for robust knowledge bases of fuzzy controllers.

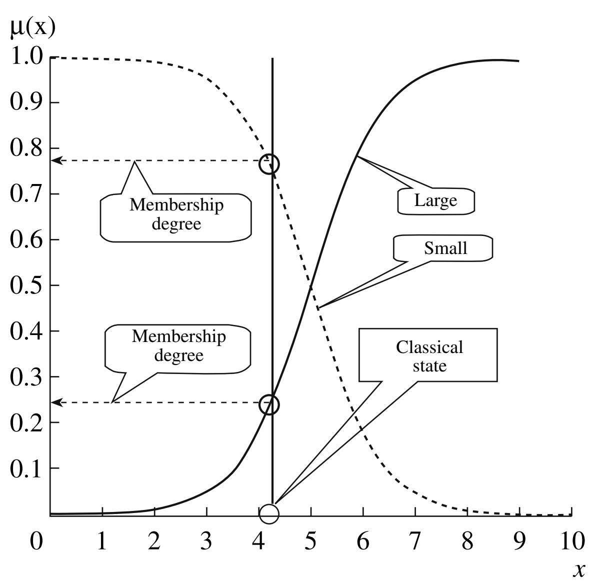


Fig. 11. Definition of fuzzy state.

In this paper, as a particular example we consider a simple case of quantum fuzzy inference presented in Fig. 1 (without optimization of the choice of the form and type of quantum correlation). The model of simplified quantum fuzzy inference and its specific features were also considered in detail in [4, 10]. The input signals in quantum fuzzy inference are converted in the superposition state $K_1(t) \otimes K_2(t)$, where $\otimes$ is the tensor product operation, and $K_{1,2}(t)$ are the gains of the PID controller formed by the knowledge bases of two fuzzy controllers (FC0 and FC1). The algorithm for calculating the superposition of control signals of different knowledge bases of fuzzy controllers was developed in [3–5] and is presented in detail in what follows. To understand the physical essence of the quantum superposition of classical states, we discuss the specific features of the definition of the quantum superposition state and its characteristic distinctions from the classical state on a fuzzy set of events.

In fuzzy sets theory, the physical state is mapped into a linguistic (in the general case, subjective) scale in the form of a linguistic variable given by an expert. Figure 11 shows the situation for the linguistic variables “large” and “small” describing the physical state in the form of a numeral measurement 4.23. According to the definition of fuzzy sets theory, we have a fuzzy state ìsmallî as 4.23/0.79 and fuzzy state ìlargeî as 4.23/0.21.

Thus, a number is characterized by two linguistic variables and can be interpreted simultaneously as ìsmallî with the membership degree 0.79 and ìlargeî with the membership degree 0.21. In this case, the linguistic variable ìlargeî is characterized by the membership function μ_L, which is defined in terms of the membership function μ_S of the linguistic variable ìsmallî in the form $\mu_L = 1 - \mu_S$, i.e., involves the negation law. Note that the law of excluded middle is not satisfied, and the fuzzy state itself is non-measurable in view of the absence measurement processes in fuzzy sets theory.

In quantum mechanics, the quantum superposition state consists of two or more classical states with given probability amplitudes and, physically, is an observable state (called observable in quantum mechanics) Figure 12 shows the quantum state of superposition of two states ìlargeî and ìsmall.î

The essential distinction of quantum superposition from a fuzzy state consists in the fact that the result of measurement of observable (in a given measurement basis) is a single classical state that have the greatest probability amplitude. All the other states in the superposition are not accessible for the observer. The quantum superposition state is objective and does not depend on the observer and is confirmed by many experiments in quantum mechanics. Note that in the quantum superposition, the principle of complementarity for numerical values of the states themselves is satisfied, which includes the law of logical negation as a particular case. For example, for the quantum state of superposition of the "large" $|1\rangle$ and "small" $|0\rangle$ in the form $\frac{1}{\sqrt{3}}(|0\rangle + \sqrt{2}|1\rangle)$, presented in Fig. 12, we have a conservation law in the form

$$\sum_{i=1,2} p_i = \left(\frac{1}{\sqrt{3}}\right)^2 + \left(\sqrt{\frac{2}{3}}\right)^2 = 1.$$

However, in contrast to the fuzzy state considered above, the linguistic variables ìlargeî and ìsmallî are characterized by different numerical values in the general case.

In quantum computing theory (see Section 2), the computations start from the action of the evolution operation U_f of the initial state $|00\ldots0\rangle$ in the form of the unitary Walsh–Hadamard transform

$$U_f = \otimes U_{f(i)},$$

where

$$U_{f(i)} = \begin{pmatrix} \sqrt{f(i)} & -\sqrt{1-f(i)} \\ \sqrt{1-f(i)} & \sqrt{f(i)} \end{pmatrix}$$

and $\sqrt{f(i)}$ determines the probability amplitude of the ith classical state in the quantum superposition. As a result, we have

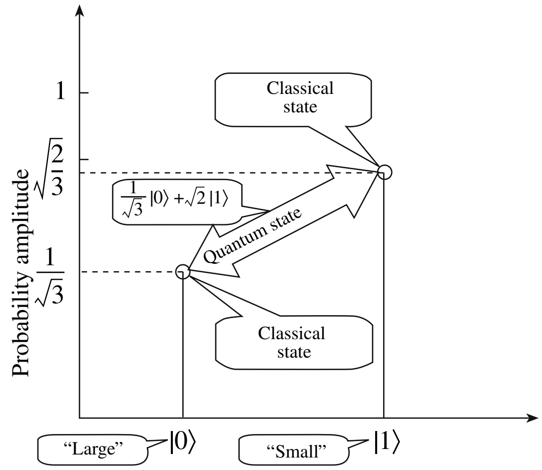


Fig. 12. Definition of the superposition quantum state.

$$U_f|00\ldots0\rangle = |s_f\rangle,$$

where $|s_f\rangle$ determines the superposition state among a finite number of classical states.

Thus, each operator $U_{f(i)}$ maps a particular quantum bit from the initial state to the mixed superposition state with a given state probability $f(i)$. The geometrical interpretation of the operator $U_{f(i)}$ is the Bloch sphere with rotation about the axis y by the angle

$$\theta_i = 2\arcsin(\sqrt{f(i)}).$$

Figures 3 and 6 shows the process of designing the quantum superposition state from classical states of measurement of the gains of the PID controller according to the scheme of Fig. 9. For example, if the normalized state $|0\rangle$ (a coded value of a gain) for the current time instant was 0.91, then the probability of this state is 0.8 (see Fig. 3). According to the law of probability conservation, the virtual complementary state $|01\rangle$ has the probability of revealing 0.2 with the numerical value 0.6 (note that $0.91 + 0.6 \neq 1$ in view of the principle of complementarity, rather than applying the logical negation law $1 - 0.91 = 0.09$).

Figure 13 shows the block of designing the quantum superposition state built-in in the main menu (see Fig. 8). Figure 14 presents the initialization of the functions of quantum fuzzy inference in the developed software product. Figure 15 presents graphically the result of initialization of the process of forming the quantum superposition state based on the Walsh–Hadamard transform.

Applying the Walsh–Hadamard transform, we organize the process of forming hidden (unobservable) mixed correlation in the superposition of signals of two classical knowledge bases of fuzzy controllers; entangled states are simulated by a quantum oracle, which can determine the maximum probability amplitude on the set of the corresponding classical superposition

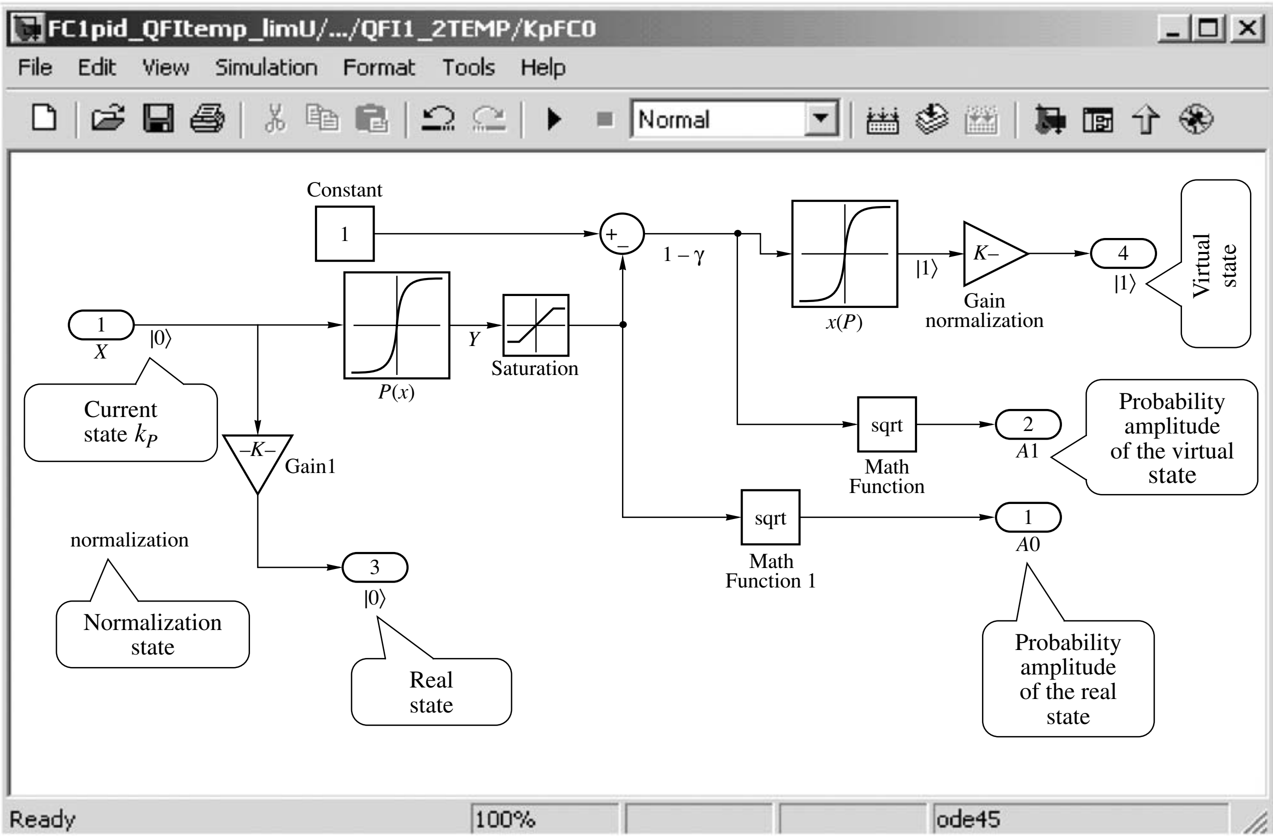


Fig. 13. The design of quantum superposition state.

states. Based on the maximum of the probability amplitude, the correlation state is extracted with the help of the interference operator (in the form of the identity operator) together with the procedure of classical measurement (observation). Figure 7 illustrates the informal content of the computational process of the quantum algorithm in quantum fuzzy inference and the functioning of quantum operators as a whole.

6.3. Specific Features of Applying Quantum Fuzzy Inference

Let us stress the fundamentally important specific feature of operation of the quantum algorithm (in the quantum fuzzy inference model) in the process of designing robust laws for controlling the gains based on the designed individual knowledge bases.

A. The robust laws designed by the model of quantum fuzzy inference are determined in a learning mode based on the output reactions of individual knowledge bases (with a fixed set of production rules) to the current unpredicted control situation in the form signals for controlling coefficient gains schedule of the PID controller and implement the adaptation process in online. This effect is achieved only by the use of the laws of quantum information theory in the developed structure of quantum fuzzy inference (see the description of four facts from quantum information theory in Section 3).

From the point of view of quantum information theory, the structure of the quantum algorithm in quantum fuzzy inference plays the role of a quantum filter simultaneously. The knowledge bases consist of logical production rules, which, based on a given control error, form the laws of the coefficient gains schedule in the employed fuzzy PID controllers. The quantum algorithm in this case allows one to extract the necessary valuable information from the reaction of two (or more) knowledge bases to an unpredicted control situation by eliminating additional redundant information in the laws of the coefficient gains schedule of the controllers employed. The output signal of quantum fuzzy inference is provided by new laws of the coefficient gains schedule of the PID controllers (see Fig. 16 in what follows). Note that for forming a robust knowledge base in on-line, any signals of learning can be the input to quantum fuzzy inference obtained from individual knowledge bases independently of the applied software simulation tools (or TS) obtained experimentally) [10].

B. At the second stage of design with application of the model of quantum fuzzy inference, we do not need yet to form new production rules. It is sufficient only to receive in on-line the reaction of production rules in the

```
%this script prepares Hystograms (probabilities)for all specified training pattern;
global Q_A_params;

% Case 4 oscillator, Example 3: beta= 0.1, Ref=0 DELAY=0.025; tC=0.05
% LIMITED control force U
filename0=['c4dex3limU_TS0_FC0', '.pat'];% Case 4 pattern file name for FC0
filename1=['C4dex3limU_TS1_FC1', '.pat'];% Case 4 pattern file name for FC1

maxKp=50; % value used for Kp normalization
maxKd=30; % value used for Kd normalization
maxKi=20; % value used for Ki normalization
% CALCULATIONS
indexes=[4 5 6];% indexes of collumns of the pattern containing Kp, Kd and Ki
%after this script, probabilities and corresponding values will be
%allocated row wise, e.g. simulink block parameter P for Kp=P(1,:), and x for Kp will be X(1,:);

pat=load(filename0);
P0=[];
X0=[];
for i=1:length(indexes);
  [p,x]=makemap(pat(:,indexes(i)));
  P0=[P0;p];
  X0=[X0;x];
end

pat=load(filename1);
P1=[];
X1=[];
for i=1:length(indexes);
  [p,x]=makemap(pat(:,indexes(i)));
  P1=[P1;p];
  X1=[X1;x];
end
Q_A_params =[1 1 1] % for initialization
% NEW FC0new  TS1 TEMP
%Q_A_params = [2.3969  2.0594  1.4125]  %gal GOOD chosen
```

File of patterns: Indices = [456]
$\{[E\ \ dE\ \ IE]\ \ \ [k_P\ \ k_D\ \ k_I]\}$

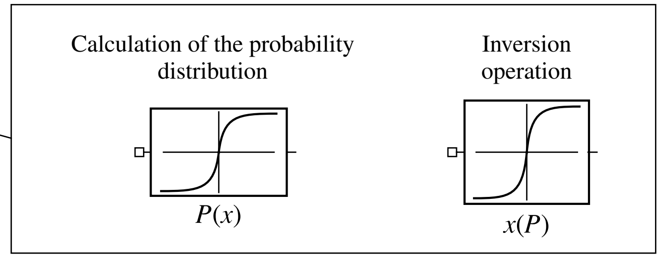


Fig. 14. The initialization of QFI functions: Matlab function (initQFI.m).

employed FC fuzzy controller to the current unpredicted control situation in the form of the output signals for controlling the coefficient gains schedule of the fuzzy PID controller. In this case, to provide the operation of the model of quantum fuzzy inference, the knowledge of particular production rules fired in the knowledge base is not required, which gives a big advantage, which is expressed the form of an opportunity of designing control processes with the required robustness level in on-line. Note that the achievement of the required robustness level in an unpredicted control situation essentially depends in a number of cases on the quality and quantity of the employed individual knowledge base.

Thus, the quantum algorithm in the model of quantum fuzzy inference is a physical prototype of production rules, implements a virtual robust knowledge base for a fuzzy PID controller in a program way (for the current unpredicted control situation), and is a problem-independent toolkit. The presented facts give an opportunity to use experimental data of the teaching signal without designing a mathematical model of the controlled object. This approach offers the challenge of using quantum fuzzy inference in problems of controlling a plant with weakly formalized structure and a large dimension of the phase space of controlled parameters. Let us illustrate the efficiency of application of quantum fuzzy inference by a particular example.

7. RESULTS OF SIMULATION OF THE OPERATION OF A SELF-ORGANIZING FUZZY CONTROLLER WITH APPLICATION OF QUANTUM FUZZY INFERENCE IN UNPREDICTED CONTROL SITUATIONS

Consider practical application of the developed model of quantum fuzzy inference for forming processes of the controlling coefficient gains schedule of a fuzzy PID controller depending on the type and form of quantum correlation. This approach is applied to simulating robust knowledge bases for fuzzy controllers in the structures of intelligent control systems for essentially nonlinear controlled object. Models of controlled objects have different forms of dynamic instability and function in various unpredicted control situations. Figure 16 shows the block diagram of the simulation of a self-organizing fuzzy controller in Simulink.

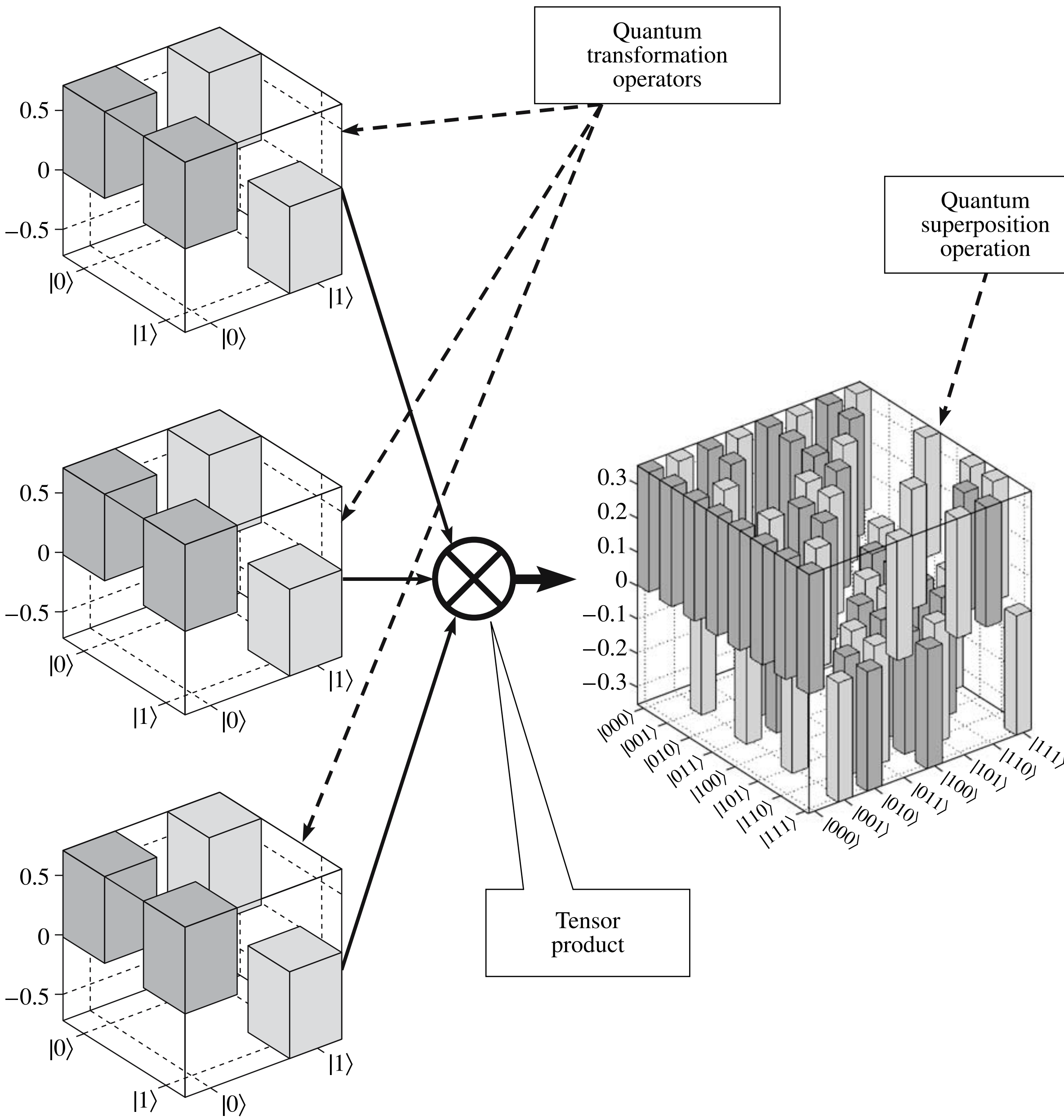


Fig. 15. The preparation of the superposition state based on the Walsh–Hadamard transform and tensor product.

In accordance with the quantum algorithm in quantum fuzzy inference, the following operations were performed at the first stage: the normalization of data of two knowledge bases (TS0 and TS1); coding of states and statistical processing in order to form an integral law of probability distribution; formation of the superposition of the corresponding coded control signals for three types of quantum correlation, spatial, temporal, and spatial–temporal.

Then, according to the next step of the quantum algorithm, amplitudes are calculated and the maximum amplitude of the probabilities of quantum states are determined (the actions of the oracle and interference) in the formed superpositions; note that for each time instant, control laws for three types of correlations were determined with application of the procedures of decoding and scaling of the gains of the fuzzy PID controller in on-line mode. Consider the important problem of extracting quantum information from classical states of the designed control laws based on the choice of a type of quantum correlation using the results of comparative analysis of their effect on the control robustness.

Example 15. The effect of types of quantum correlation on the form of processes of the controlling coefficient gains schedule of a fuzzy PID controller. By

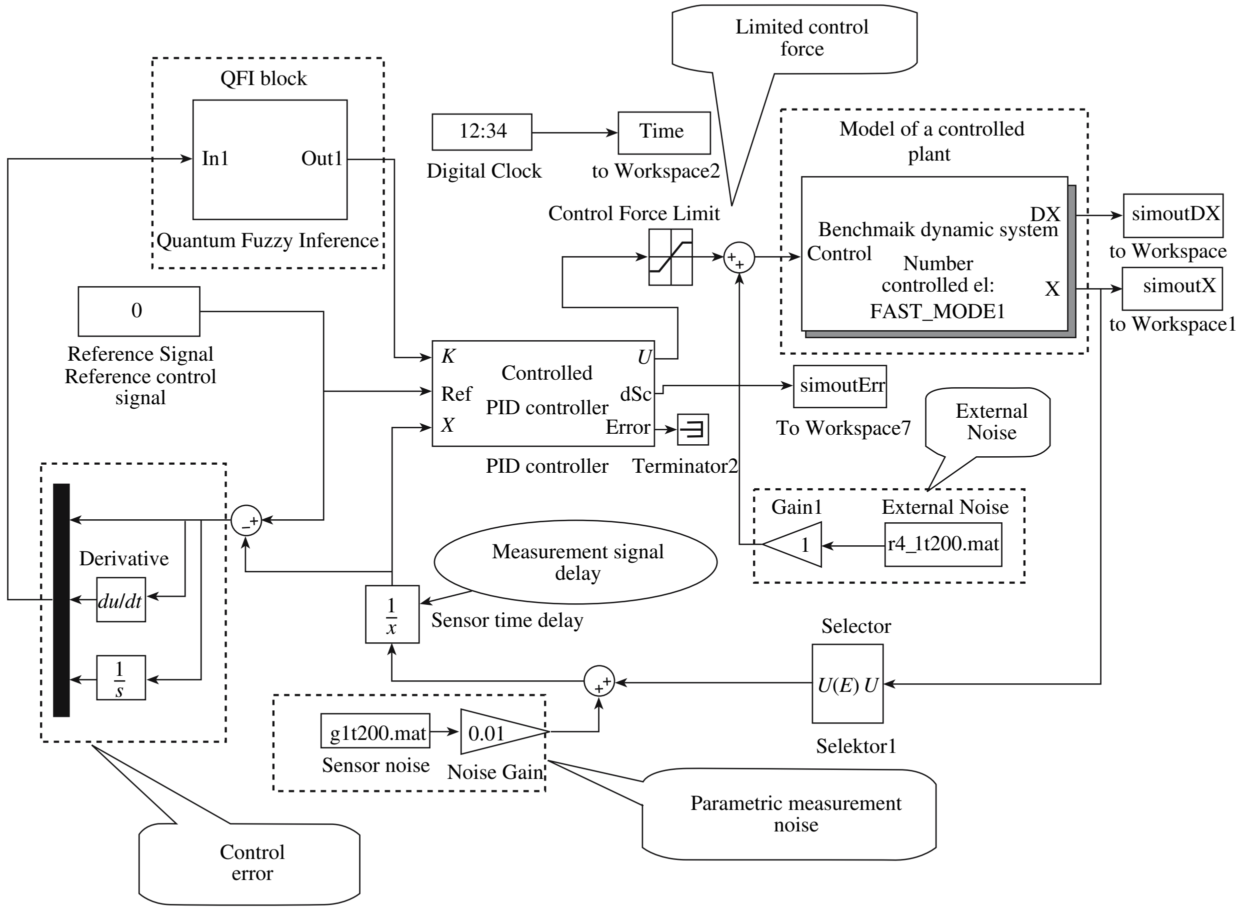


Fig. 16. The block diagram of simulation of a self-organizing FC in Simulink.

Fig. 4, we have three types of quantum correlation, spatial, temporal, and spatial–temporal. As the initial knowledge bases, we use signals for the controlling coefficient gains schedule of two fuzzy PID controllers designed by the knowledge base optimizer (see the results of simulation of the corresponding control signals in [7] and example 17 in what follows).

7.1. Sensitivity of Designed Individual Knowledge Bases

First of all, we consider the sensitivity of the designed knowledge bases on the teaching situations) to the variation of the conditions of a control situation based on simulation schemes of Figs. 9 and 16. As models of a controlled object, model (7.1) described above was used. The sensitivity of a knowledge base determines the reaction of the controlled object to an unpredicted control situation. These two information sources together give a physical resource for extracting additional hidden quantum information in classical states of the variation of the coefficient gains of the PID controller.

Figure 17a shows the laws for the controlling coefficient gains schedule of two fuzzy PID controllers designed for the following situations of control in the learning mode (off-line) on the teaching signal from the knowledge base optimizer: (i) under the action of the Gaussian noise on the controlled object (teaching situation TS0); and (ii) in the condition of the action of the Rayleigh noise on the controlled object (teaching situation TS1).

Control laws were obtained based on the simulation block diagram presented in Fig. 16. All the characteristics of the random noise were described in [1, 7], and, in this section, they are used as the initial data. The noise has the same correlation function

$$R(\tau) = \sigma^2 \exp\{-\alpha|\tau|\}\left(\cos\omega\tau + \frac{\alpha}{\omega}\sin\omega|\tau|\right),$$

but differs in the functions of probability distribution (Fig. 18). The form and parameters of the forming filters are presented in [1, 7]. The effect of the noise form on the variation of the law of the controlling coefficient gains schedule for the second situation is considered in Fig. 17b (the effect of the Gaussian noise in the control situation TS1, when the knowledge base for the fuzzy

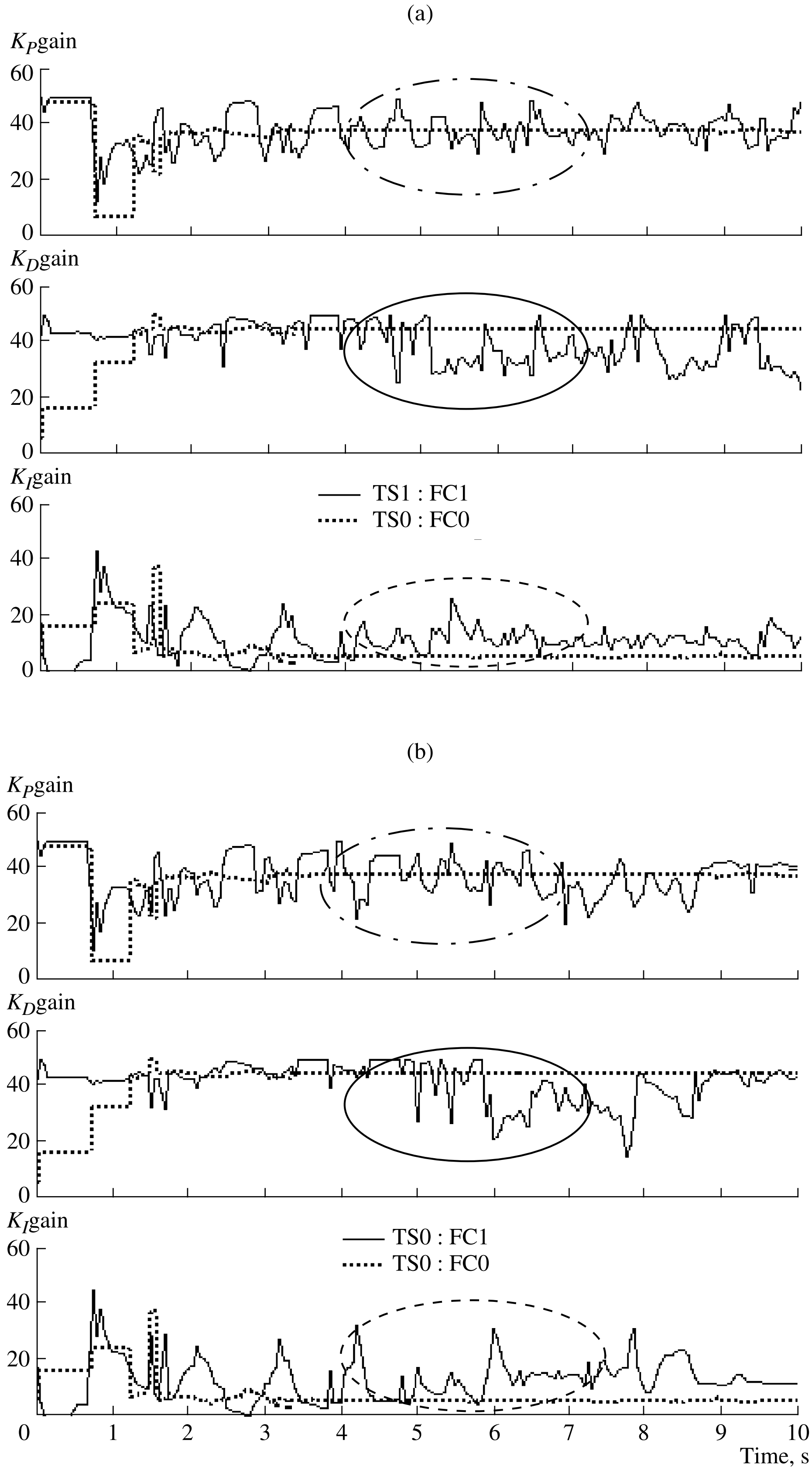


Fig. 17. The effect of the form of noise on the laws of processes of controlling the coefficient gains of fuzzy PID controllers.

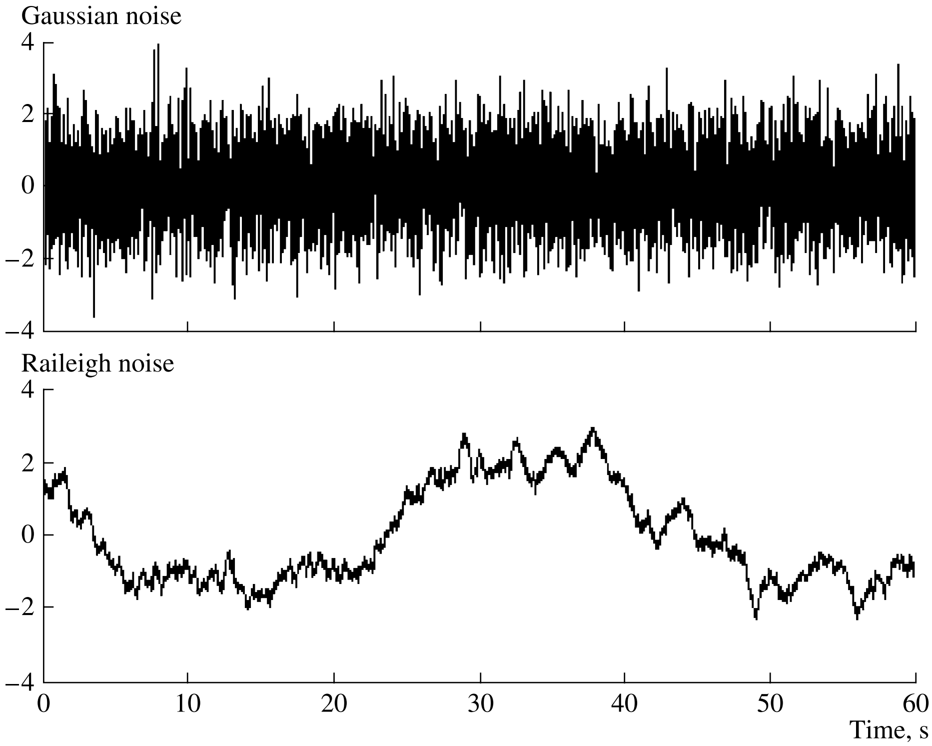


Fig. 18. The form of random noise, Gaussian noise (at the top) and Rayleigh noise (at the bottom) with the same correlation functions.

controller was designed based on the Rayleigh noise). Figures 17a and 17b specify the typical changes of the laws of the controlling coefficient gains schedule for the situation TS1 under the change of the type of random noise.

7.2. Simulation of the Types of Quantum Correlation and Control Laws

Figure 19 shows the results of simulation of the laws of the controlling coefficient gains schedule of the fuzzy PID control based on the software support and tools (Fig. 8, the menu for the choice of the correlation type) for the chosen three types of quantum correlation and the block diagram in Fig. 16.

The analysis of simulation results in Fig. 19 obtained using quantum fuzzy inference reveals the strict trend of forming simple (for software–hardware implementation) laws of the controlling coefficient gains schedule of the fuzzy PID controller for all three types of quantum correlation. The efficiency and choice of the final control law is determined by the chosen criterion of optimal control and dynamic behavior of the controlled object (see example 17 in what follows).

Figure 20 presents the result of comparison of the control quality for the three investigated types of quantum correlation according to the criterion of minimum control error. The quantum temporal correlation provides the best control quality and is used in the further simulation and investigation of the robust control in example 17. Consider the design of a self-organizing robust fuzzy controller based on the application of quantum fuzzy inference for a nonlinear controlled object in unpredicted control situations.

Example 16. Robust intelligent control of a nonlinear controlled object (with essential dissipation and local instability). The motion equations of the controlled object and the formation of the control force are considered together with the thermodynamic conditions in the following form:

$$\ddot{x} + [2\beta + a\dot{x}^2 + k_1 x^2 - 1]\dot{x} + kx = \xi(t) + u(t);$$
$$\frac{dS_x}{dt} = [2\beta + a\dot{x}^2 + k_1 x^2 - 1]\dot{x}\dot{x}, \quad (7.1)$$

where $\xi(t)$ describes random perturbations with a given density function of the probability distribution; $u(t)$ specifies the desired optimal control force; and S_x represents the entropy production in the controlled object.

The physical (rheological) model for (7.1) is the electromechanical part of the suspension of a moving object (vehicle, motor cycle, railway carriage, etc.) with essentially nonlinear dissipation for improvement of the effect of damping the oscillations, which is described by the generalized Duffing–Van der Pol (Halmos–Render) equation. Under the condition $2\beta + a\dot{x}^2 +$

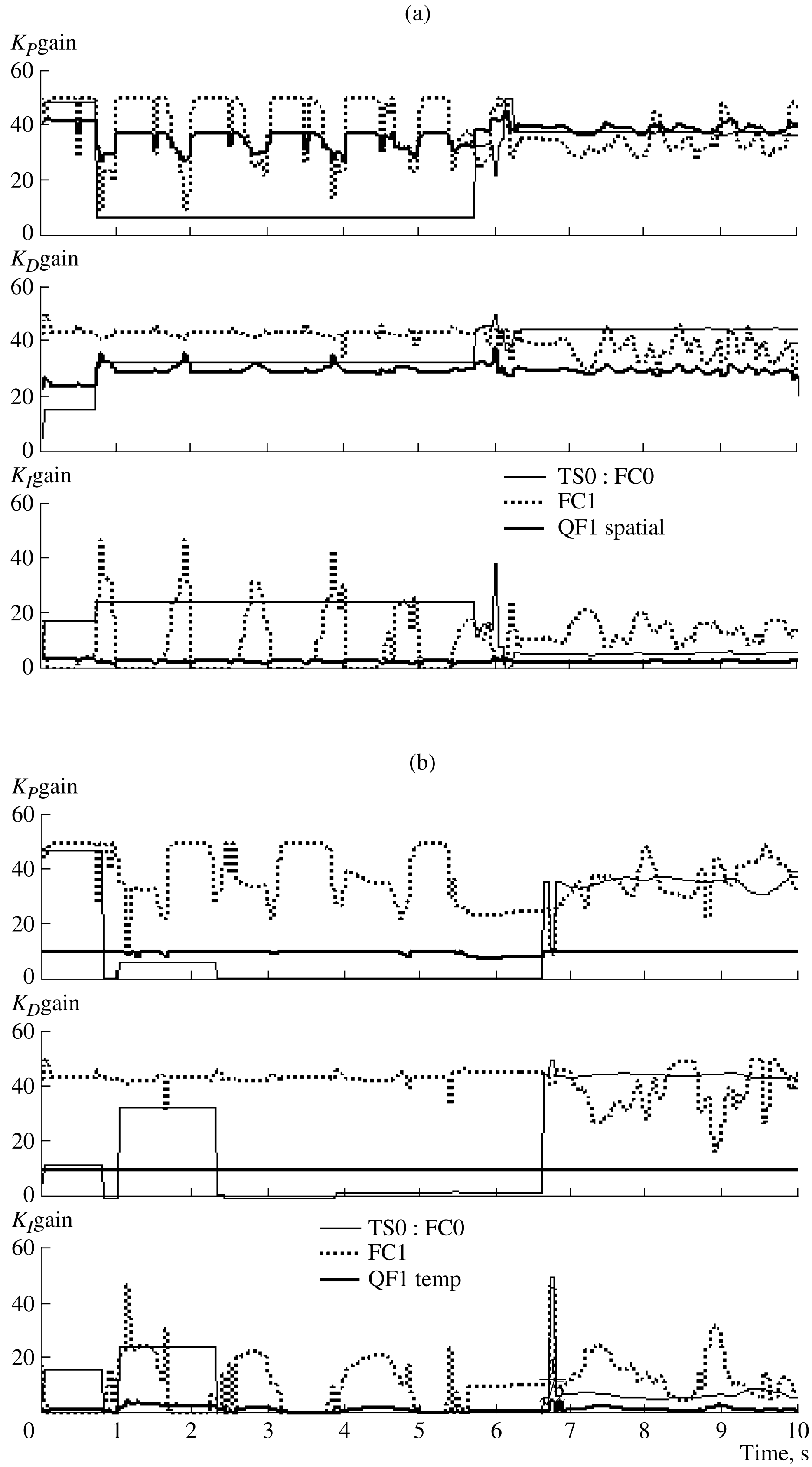


Fig. 19. The effect of types of quantum correlations on the form of laws for controlling the coefficient gains of fuzzy PID controller.

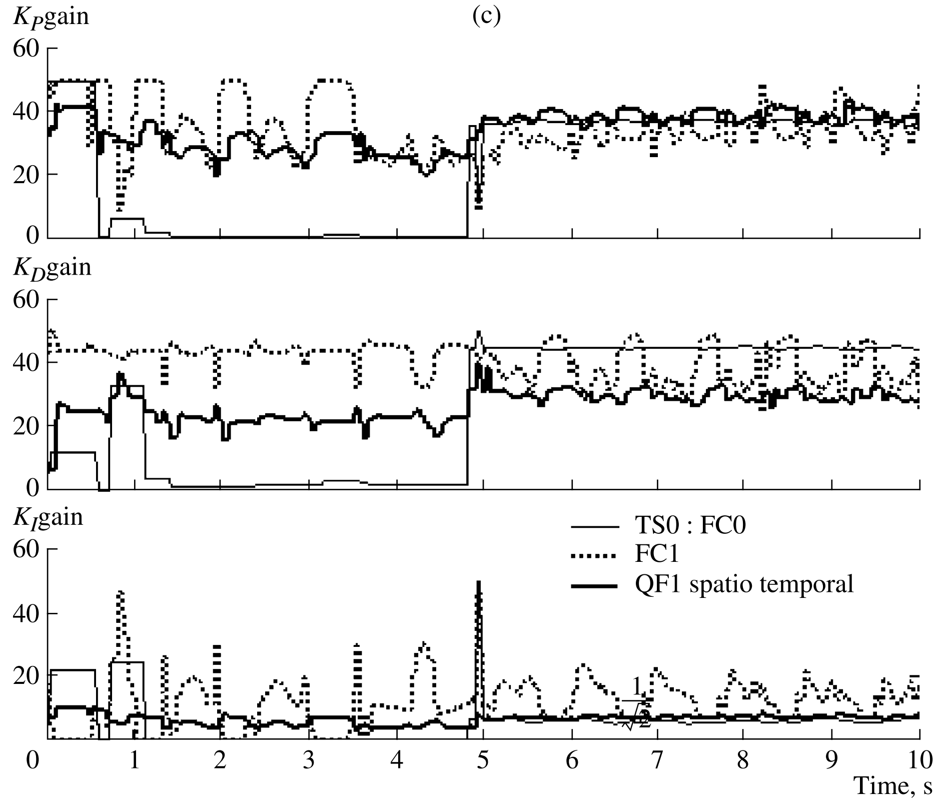


Fig. 19. Contd.

$k_1x^2 \leq 1$, the system is locally unstable, and the entropy production rate is negative, i.e., $\frac{dS_x}{dt} \leq 0$.

System (7.1) demonstrates different dynamic behavior when the structural parameters are changed. If $\beta = 0.5$ (the other parameters, e.g., $\alpha = 0.3$; $k_1 = 0.2$, and $k = 5$), then the system is asymptotically stable; when $\beta = -1$ (the other parameters are the same as above), the system locally unstable and has a self-oscillation regime. Dynamic system (7.1) is characterized in the phase portrait by a domain of attraction (strange attractor).

Figure 21 shows free and forced oscillations, as well as the corresponding phase portraits of the considered system under the specified parameters. Note the specific feature of the control conditions of the controlled object. For small oscillations, system (7.1) is locally unstable, which present problems in designing a robust control for this system under the reference signal (control goal) of type $x = 0$ and the presence of small random perturbations. Figures 22a and 22b present the entropy production and entropy production rate in the dynamic behavior of the controlled object (free-oscillation mode) depending on the variation of the parameter β in (7.1).

***A**. Consider the following teaching situations.*

Teaching situation for designing the knowledge base of fuzzy controller 1, FC1. As the source parameters of system (7.1), we take $\beta = 0.5$; $\alpha = 0.3$; $k_1 = 0.2$, and $k = 5$ and the initial conditions equal to x_0; $\dot{x}_0 = 2.5$, 0.1, the reference signal $x_{ref} = 0$, the range of variation of the gains [0, 10], and the external perturbation is a random action with the Rayleigh law of probability distribution. This control situation is denoted by TS1.

Teaching situation for designing the knowledge base of fuzzy controller 2, FC2. As the source parameters of system (7.1), we take $\beta = -1$; $\alpha = 0.3$; $k_1 = 0.2$, and $k = 5$, the initial conditions equal to x_0; $\dot{x}_0 = 2.5$, 0.1, the reference signal $x_{ref} = -1$, the range of variation of the gains [0, 10], and the external perturbation (as in TS1) is a random action with the Rayleigh law of probability distribution. This control situation is denoted by TS2.

The knowledge bases for these teaching situations (TS1 and TS2) were designed in [1, 7]. A random signal affecting system (7.1) with the Rayleigh law of probability distribution was simulated (see Fig. 18) with the help of the nonlinear forming filter described in [7]. Using the tools of the knowledge base optimizer and a teaching signal obtained by stochastic simulation with the genetic algorithm from [7], for the given teaching situations, we designed knowledge base 1 (KB1) and knowledge base 2 (KB2) for fuzzy controller 1 (FC1)

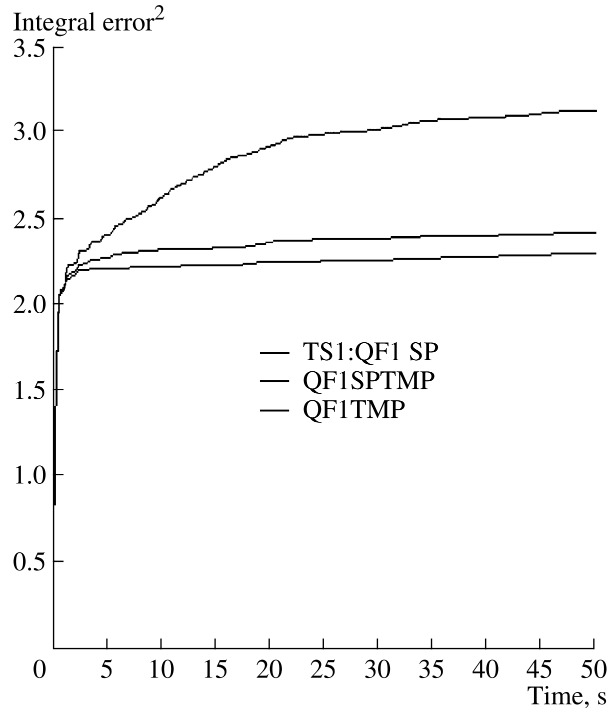


Fig. 20. The comparison of the control quality according to the minimum control error criterion for three types of quantum correlation (in learning situation).

and fuzzy controller 2 (FC2) under the action of the Rayleigh noise that approximated optimally the reference signal (from the point of view of the taken fitness function and the given control situations for the controlled object).

The specific features of designing a knowledge base based on the knowledge base optimizer. For more complete understanding of the results of simulation and design of fuzzy controllers, we note certain specific features of the software support of the knowledge base optimizer (described preliminarily in Section 5) and the technical characteristics of the knowledge base of fuzzy controller designed based on it, and we also compare these results with the results of design obtained using the FNN tools of type AFM.

Figure 23 presents the general structure of the simulation of the main blocks of the fuzzy controller and shows the structure of the main blocks of the software support of the design of knowledge bases. Figure 23c shows an example of a block for fuzzy inference using the Sugeno model built-in in the knowledge base optimizer. Structurally, the knowledge base optimizer has a program interface with Matlab and is connected by a program of the quantum fuzzy inference block via the built-in interface (see Fig. 8).

Let us discuss in short the process of designing the teaching signal of optimal control as an output signal of the knowledge base in the fuzzy controller. As the teaching situation, we take the situation TS1. In this case, the cost function of the control in the form of control error minimum is regarded as the fitness function of the genetic algorithm with the search space for the gains (0, 5). To design a knowledge base in this control situation, three input variables $\{e, \dot{e}, \int e dt\}$, describing the dynamic behavior of the control error and three output variables $\{k_p, k_d, k_i\}$, representing the gains of the designed fuzzy PID controller, are given.

In the knowledge base optimizer, the process of designing knowledge bases yielded the following characteristics: (1) the number of membership functions for each output linguistic variable (7, 9, 9) (determined optimally by GA1), respectively; (2) the total number of production fuzzy rules $7 \times 9 \times 9 = 567$; (3) the optimal number of production rules chosen for the knowledge base according to the criterion of frequency of inquiries from the knowledge base was 20; and (4) the optimal number of production rules selected by GA2 was 20.

Figure 24 presents the optimal form of the membership function for the third output variable k_I (the gain of the integral error).

For the comparison, we present in Fig. 25 the results of operation of FNN of type AFM. For each linguistic variable, we have five membership functions given manually, the total number of production fuzzy rules was 125, and the number of activated rules was also 125. In the AFM, the number and form of the membership function are determined by an expert, while, in the knowledge base optimizer, all design operations are automated in an optimal way based on a genetic algorithm. The analysis of results has shown that the structure of the knowledge base with the specified parameters designed based on the knowledge base optimizer provides a higher robustness level for the fuzzy controller compared with the FNN (125 rules) and a PID controller with constant parameters (5, 5, 5).

B. *Let us formulate the following control problems*:

(1) steer the controlled object from the initial state $x_0, \dot{x}_0 = 2.5, 0.1$ to a final state (representing the reference signal (RS)) under external noise acting on the controlled object and under the presence of variations of the parameters of the model of the controlled object and control goals (RS);

(2) estimate the sensitivity and robustness level of the designed fuzzy controllers (1, 2) and compare with the conventional PID controller and the self-organizing controller with a database obtained based on the application of quantum fuzzy inference in an unpredicted control situation.

Figure 26 presents the results of simulation of control processes in the specified conditions for the situation TS2, which is unpredicted for FC1 (the parameter of the structure of the controlled object and the control goal were essentially changed). The simulation results are analyzed in what follows. Figure 27a present the

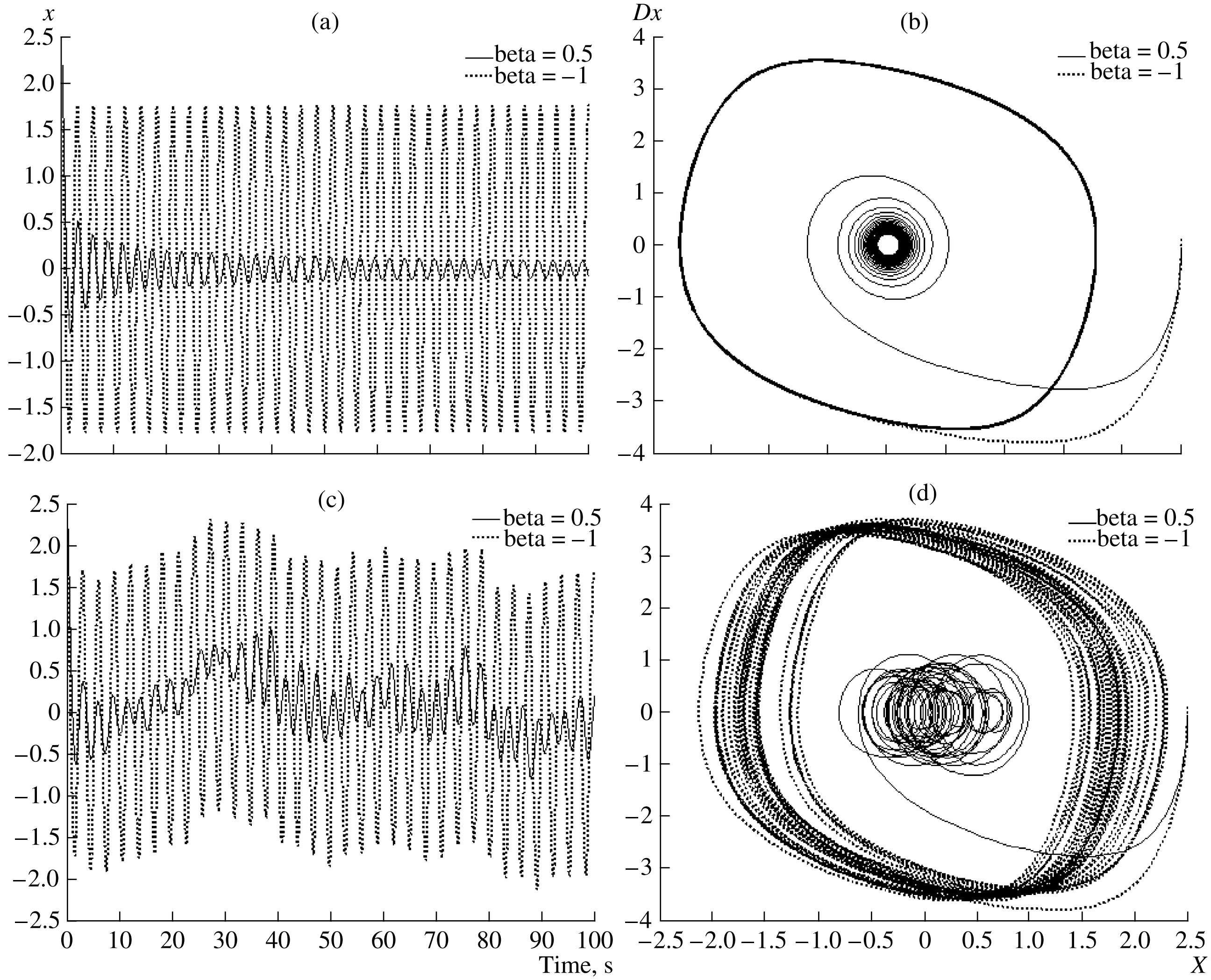


Fig. 21. The dynamic behavior of the CP: (a) free oscillations; (b) the phase portrait of free oscillations; (c) forced oscillations; and (d) the phase portrait of forced oscillations.

Rayleigh noise used in simulation, and Fig. 27b shows the laws for controlling the gains obtained by using quantum fuzzy inference, when the reactions of the knowledge bases of the two fuzzy controllers FC(1, 2) to unpredicted control situation are used.

C. *Analysis of the simulation results and physical interpretation of the self-organization process in fuzzy controllers.* Figure 26a reflects the main advantages and the operation of the self-organization principle. At the stage of the transient process, the self-organizing fuzzy controller qualitatively follows along the trajectory of the conventional PID controller, but with a reduced integral gain (see Fig. 27b). Because of this, the resource expenditure in the form of useful work of the PID controller is reduced (the value of the produced entropy). At this stage, the motion of the controlled object itself is used (at the stage of the transient process up to the reaching the control goal by the controlled object, the coefficient gains of the self-organizing PID controller have constant values).

Figure 28 presents the values of generalized entropies of the system "CO + FC" calculated in accordance with (7.1). According to [1, 6, 7], the necessary relations between the qualitative and quantitative definitions of the Lyapunov stability, controllability, and robustness of control processes of a given controlled object are correctly established. Before the achievement of the control goal (the reference control signal equal –1 in this case) the process of self-learning the fuzzy controller and extraction of valuable information from the results of reactions of the two Fuzzy controllers to an unpredicted control situation in on-line with the help of quantum correlation is implemented. Since quantum correlation contains information about the current values of the corresponding gains, the self-organizing fuzzy controller uses for achievement of the control goal the advantage of performance of the FC2 and

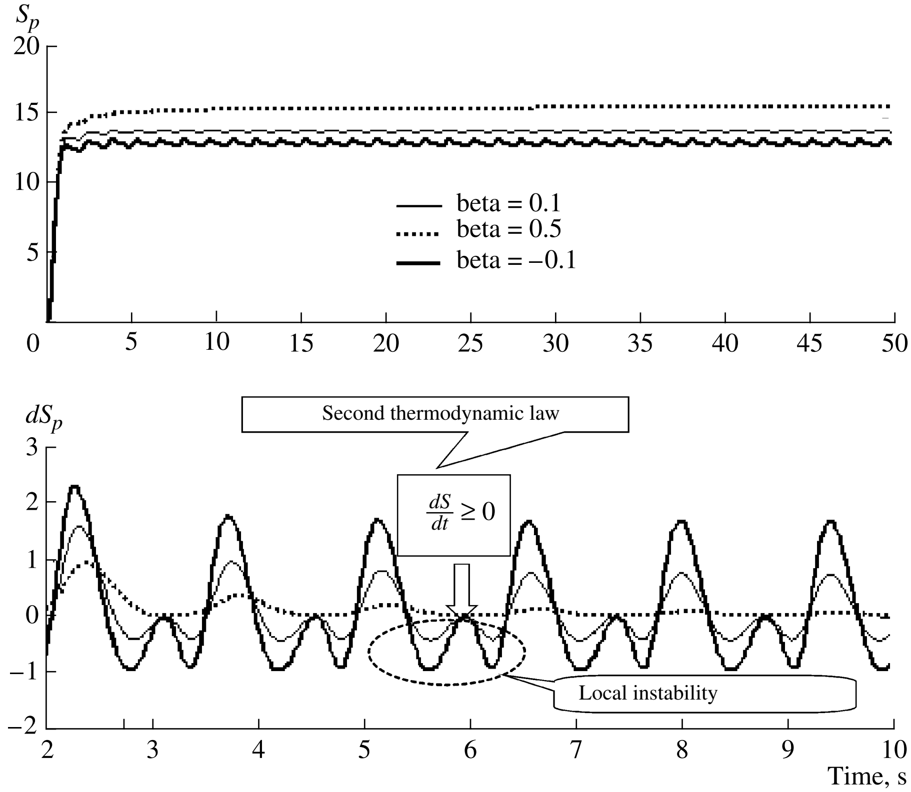


Fig. 22. The thermodynamic interpretation of the local instability phenomenon.

the aperiodic character of the dynamic behavior of the FC1. As a consequence, improved control quality is ensured.

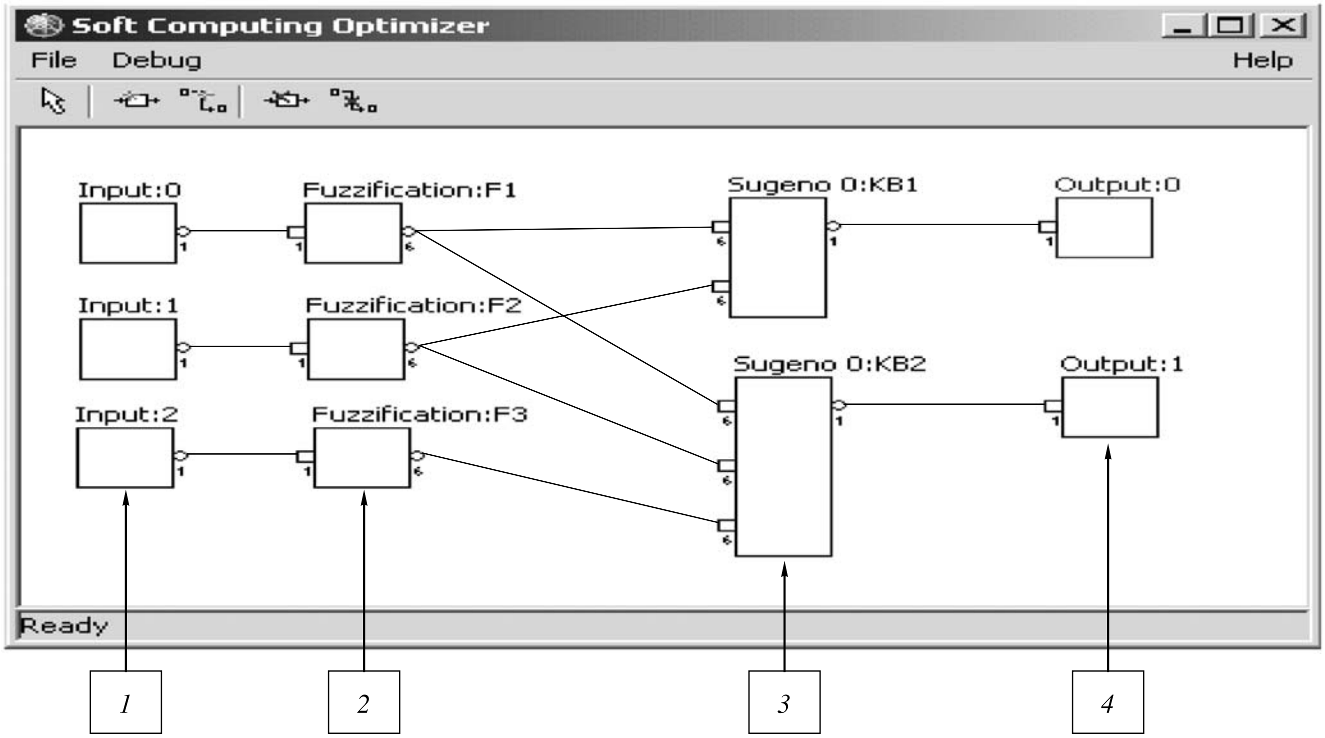
Recall that for the considered case, the FC2 was trained on the control situation TS2, and for the FC1, the situation TS2 is unpredicted (the parameter of the structure of the controlled object and the control goal were changed essentially). Therefore, using quantum correlation, we exercise coordination between the corresponding control laws. Because of this effect, the self-learning fuzzy controller extracted valuable information about the overshoot of the FC2 and the reaction of the FC1 to an unpredicted control situation, used the data on performance of the FC2 in the laws for controlling the coefficient gains (Fig. 26a), and passed to the self-adaptation regime saving the advantage of the aperiodic process of the FC1 (achievement of the control goal without overshoot). We stress that the FC1 itself does not achieve the control goal in this situation (Fig. 21), while the FC2 is overshot and is locally unstable (Fig. 28).

Example 17. Robust control in an unpredicted control situation. In example 16, the evaluation of robustness of the intelligent control system on teaching situations, when one of the situations, the TS2, was unpredicted for the first fuzzy controller (the parameter β (in the structure of the controlled object) and the control goal changed essentially), trained on the TS1 situation. Despite the paradoxical fact that the FC1 itself does not achieve the control goal in this situation (see Fig. 21), and the use of the FC2 results in overshoot and the locally unstable motion of the controlled object (see Fig. 23), the self-organizing FC fuzzy controller with application of quantum fuzzy inference coped with the control problem successfully.

We pose the problem whether this effect of application of quantum fuzzy inference retains in more complex (than on teaching situations) unpredicted control situations. To answer the posed question, we consider a more complex unpredicted (for the FC1 and FC2 simultaneously) control situation.

To create a new unpredicted control situation, we introduce the following new forming parameters in the block diagram in Fig. 16: (1) control reference signal (goal) $x_{ref} = 0$; (2) the Rayleigh external noise (Fig. 27a); (3) a new parameter $\beta = -0.1$ in the model of the controlled object; (4) a constraint on the control force $u \leq |10|[N]$; (5) a Gaussian random noise (Fig. 18) with the gain equal to 0.02; and (6) a time delay of the control error signal equal to 0.0125 s. The other parameters of the controlled object remain the same. Thus, the unpredicted control situation contains noise in the sensor system, a delay for the FC1 and FC2 in information about the control error, and jump changes of the parameter in the structure of the unpredicted.

(a)



(b)

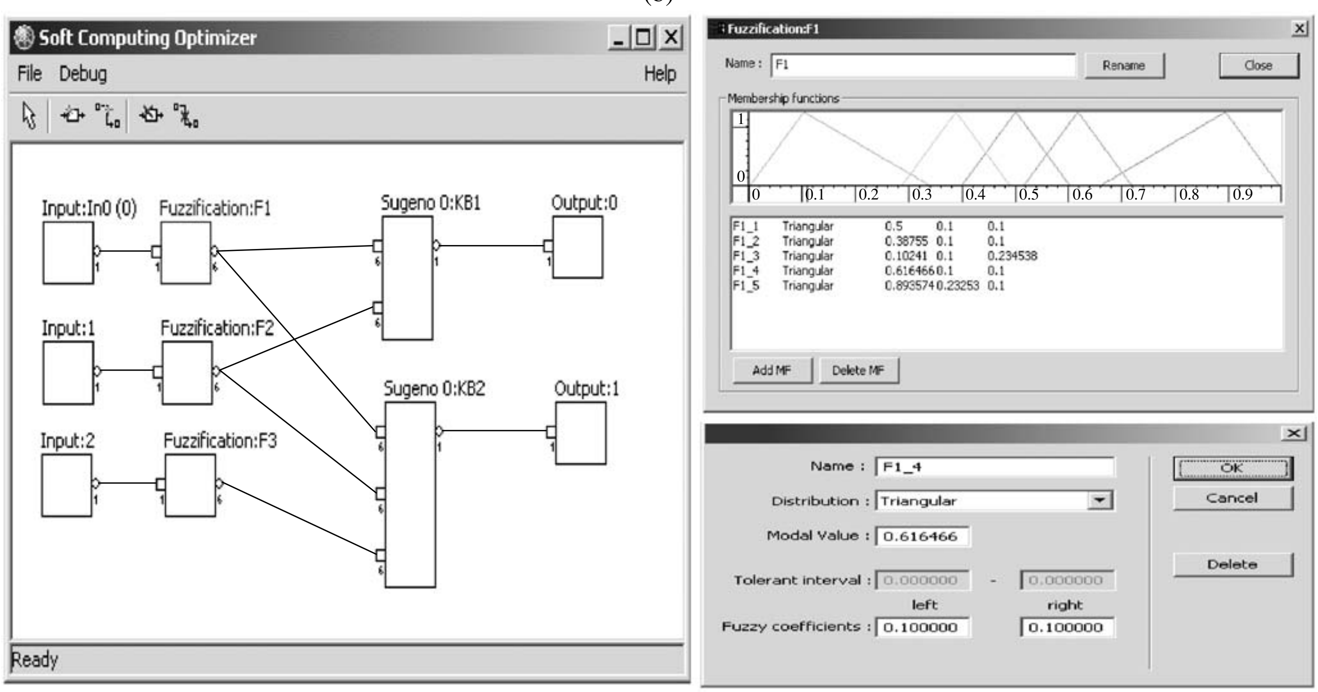
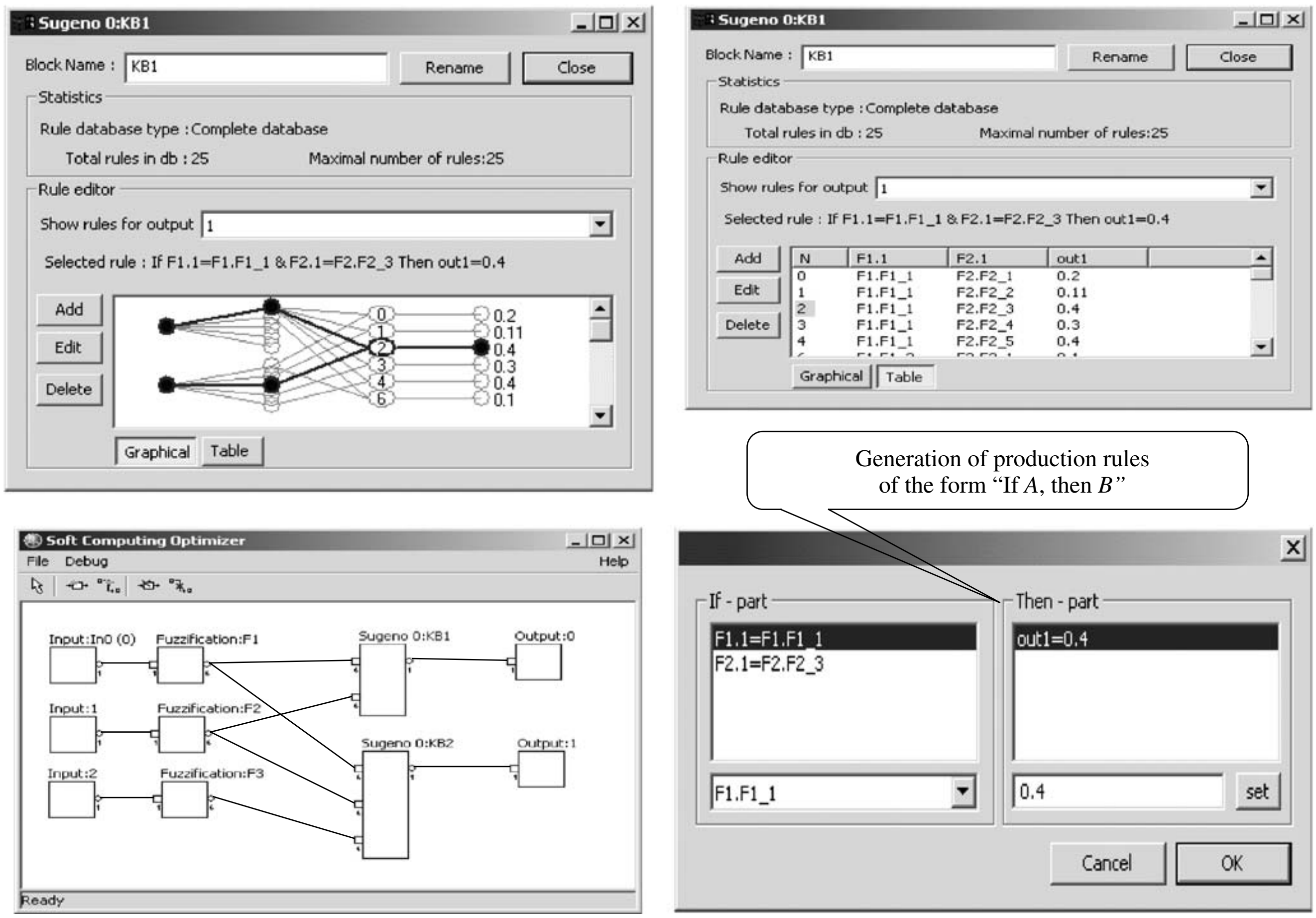


Fig. 23. The structure of the software support of designing KBs in a FC. (a) The structure of the designed FC: 1, the input port, 2, fuzzifier, 3, fuzzy inference and a defuzzifier block, 4, the output port; (b) fuzzifier; (c) fuzzy inference block.

(c)



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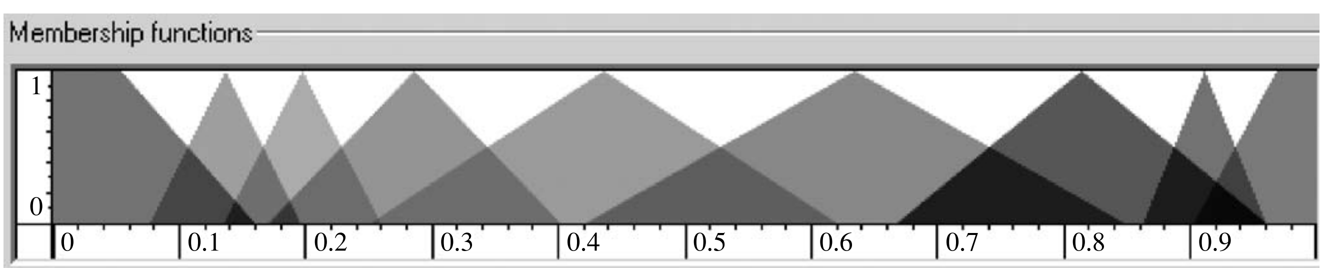

Fig. 24. The optimal form of the membership function for describing the third output linguistic variable.

Figure 29 shows the result of simulation of the dynamic behavior of control error, and Fig. 30 presents the result of simulation of the integral squared control error. The obtained control laws for variation of the coefficient gains and control force for given physical constraints are presented in Figs. 31 and 32, respectively. Figure 33 reflects the resource loss of the intelligent control system in the form of the increasing of the integral generalized entropy in the system "CO + fuzzy PID controller".

Analysis of the results presented has shown that both the FC1 and FC2 do not cope with the control problem posed and have large resource expenditures. In contrast to these fuzzy controllers, using quantum fuzzy inference, the self-organizing fuzzy controller extracted from the dynamic behavior of the controlled object, the FC1, and FC2 valuable hidden quantum

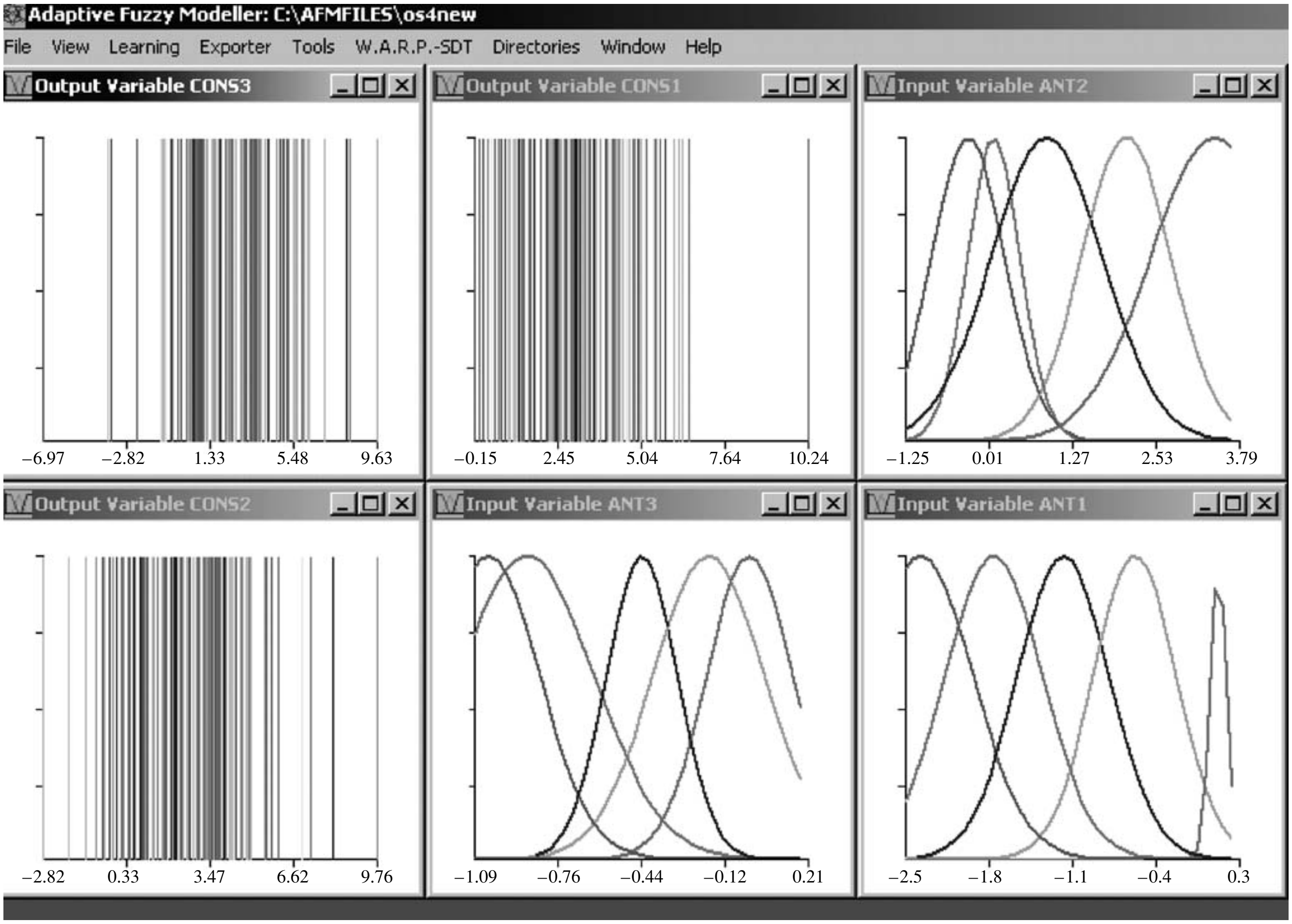

Fig. 25. An example of the choice of a membership function in FNN.

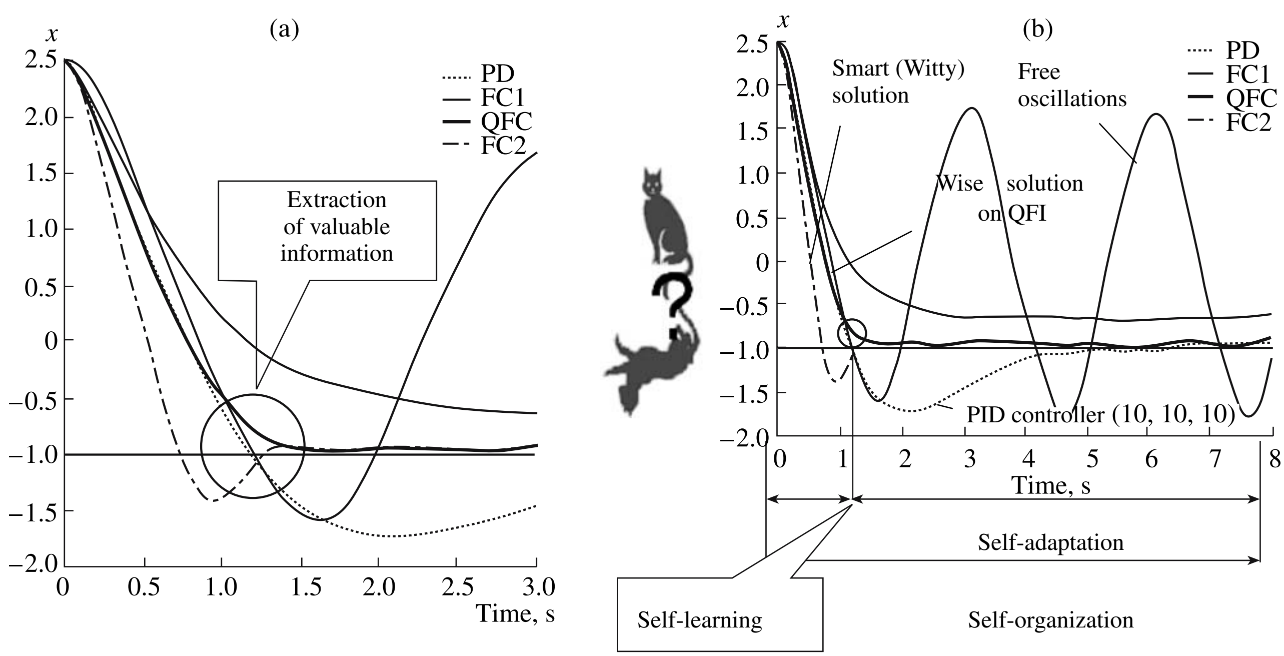

Fig. 26. The results of simulation of robust control for a nonlinear CP with a self-organizing FC in an unpredicted control situation: (a) formation of the learning process for an intelligent FC; (b) formation of the self-adaptation process of an intelligent FC.

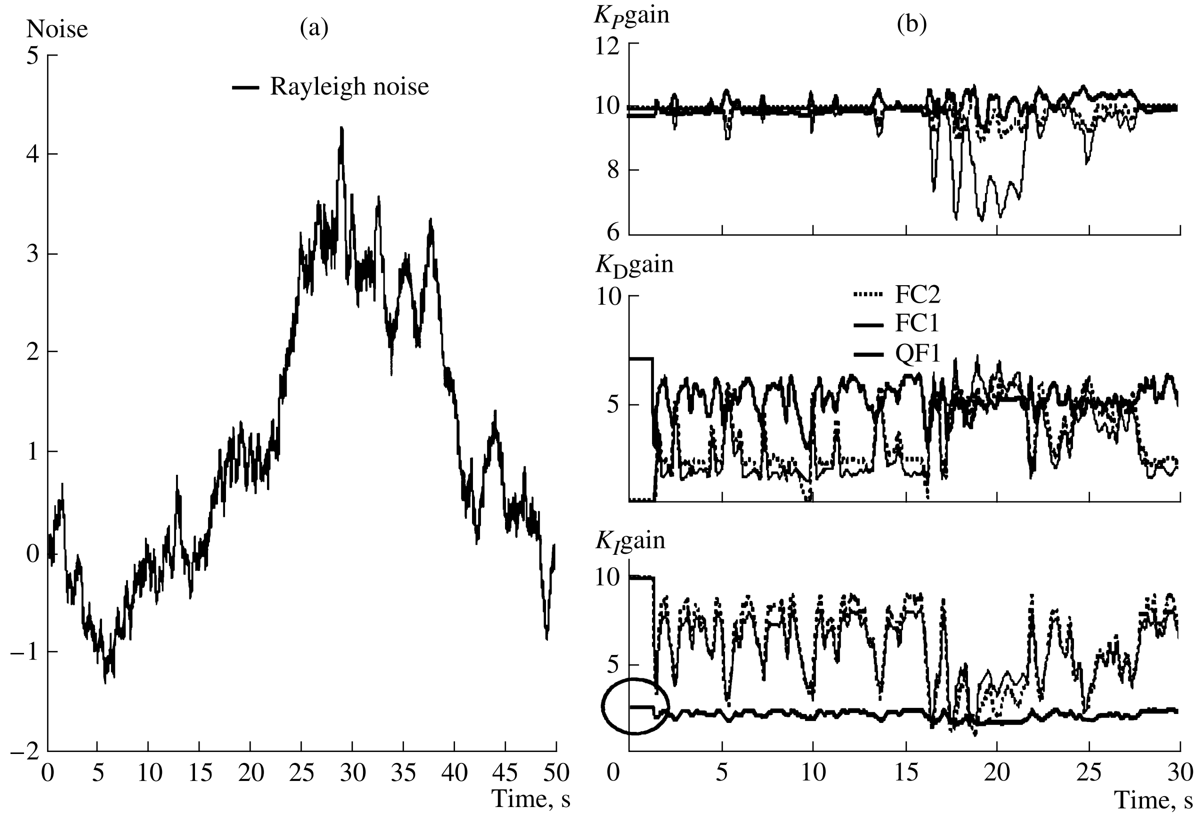

Fig. 27. The forms of a random process with a Rayleigh probability distribution function (a) and of laws for controlling the coefficient gains of a fuzzy PID controller (b).

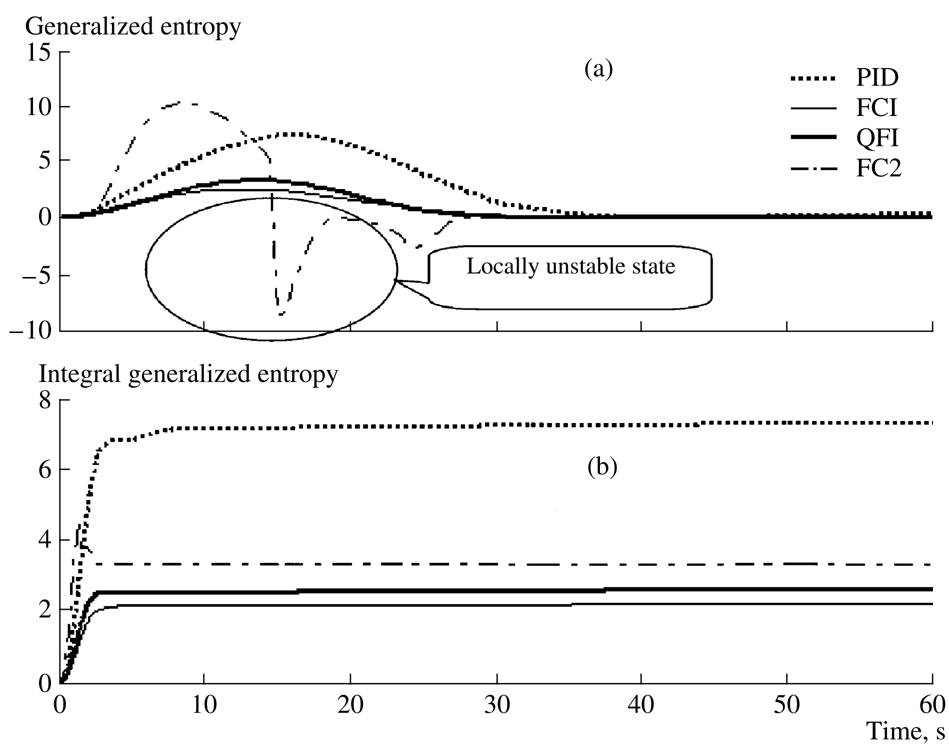

Fig. 28. The dynamic behavior of the generalized entropies (CO + FC): (a) temporal generalized entropy; (b) the accumulated value of the generalized entropy.

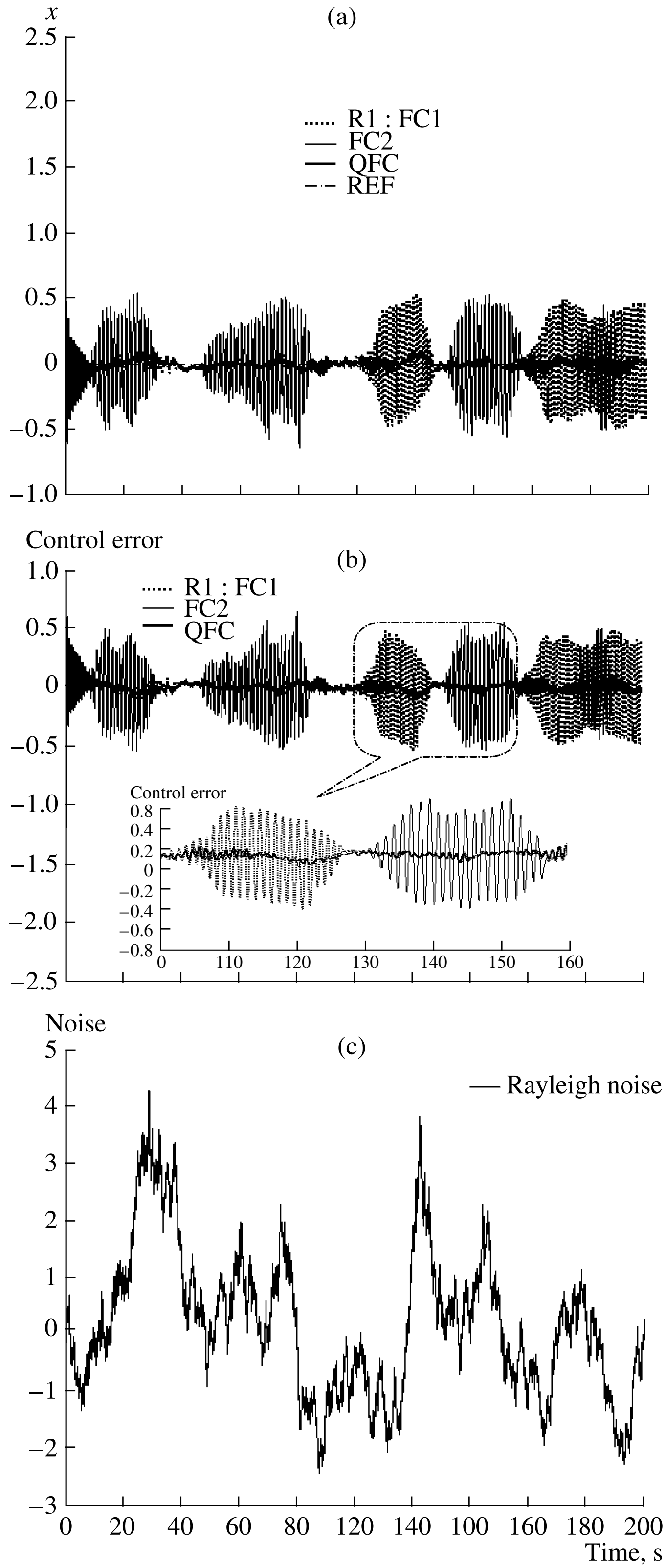

Fig. 29. The evaluation of robustness in a new control situation: (a) system motion; (b) control error; and (c) Rayleigh noise.

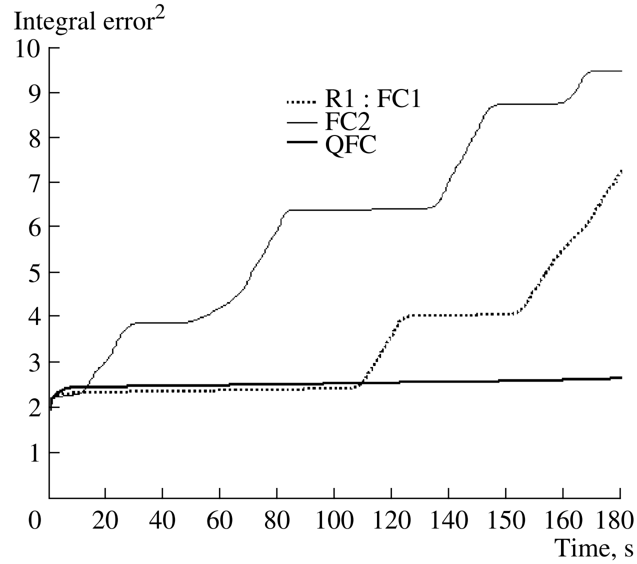

Fig. 30. Robustness evaluation in a new control situation: the integral and squared control errors.

information and achieved the control goal with the least resource loss. Thus, in complex unpredicted control situations, the effect of application of the model of quantum fuzzy inference retains, which is the answer to the posed question.

We stress that examples 17 and 18 only illustrate the capabilities of application of the developed model of quantum fuzzy inference in problems of designing intelligent control systems with an improved robustness level in unpredicted control situations. More detailed analysis of the efficiency of application of the model of quantum fuzzy inference requires an independent discussion and is not considered in this paper.

Note only the top-priority problems of the investigation of the efficiency of application of the developed model of quantum fuzzy inference, such as controlled objects with global dynamic instability, with various types of instability in parts of the generalized coordinates (with local and global instability), different combinations of internal and external noise, variation of the parameters of the structure of the controlled object, an estimate of resource loss, complexity of processing and physical implementation of control laws, as well as many other related problems. The simulation results have shown the advantage of quantum control strategies based on quantum fuzzy inference, which allow one to design a universal robust fuzzy controller from two non-robust fuzzy controllers (1, 2) with simple laws for controlling the coefficient gains of a PID controller.

Thus, the self-organization process can be divided conditionally into complementary self-learning and self-adaptation processes, improving the robustness level of intelligent control processes in on-line.

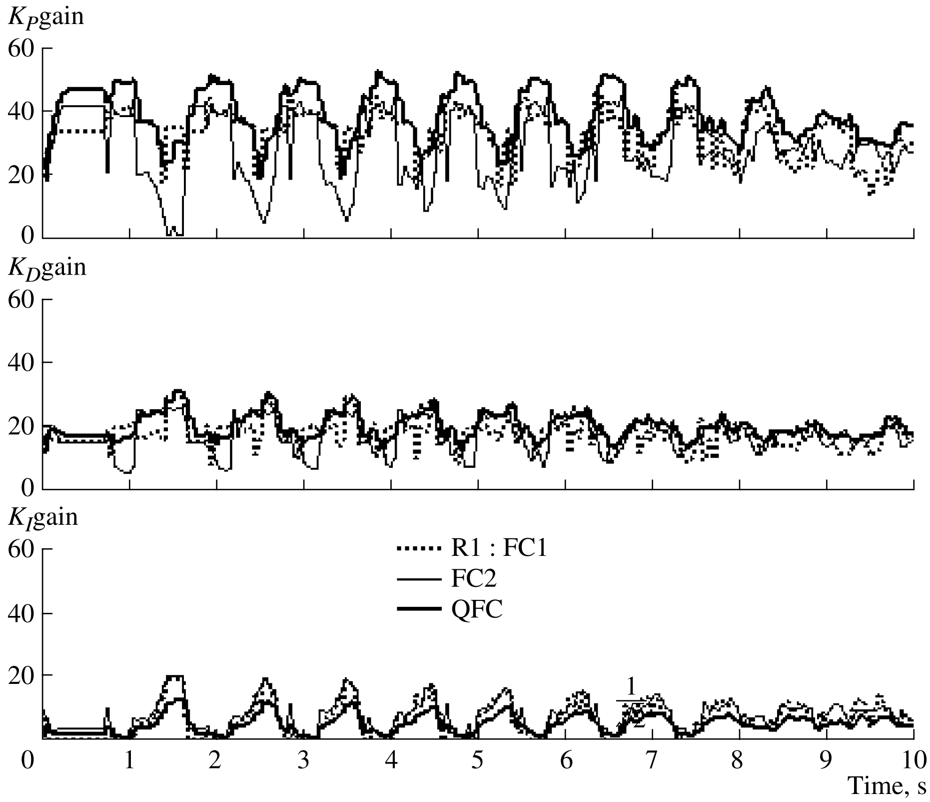

Fig. 31. Robustness evaluation in a new control situation: self-organization of laws for controlling the coefficient gains of a quantum fuzzy PID controller.

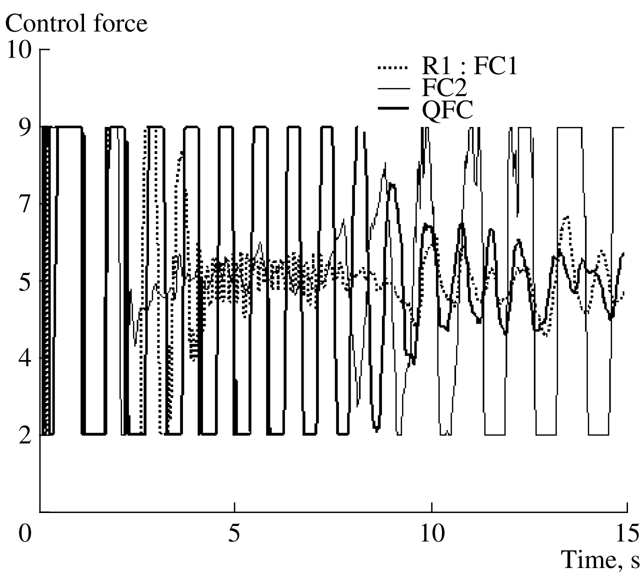

Fig. 32. Robustness evaluation in a new control situation: control force.

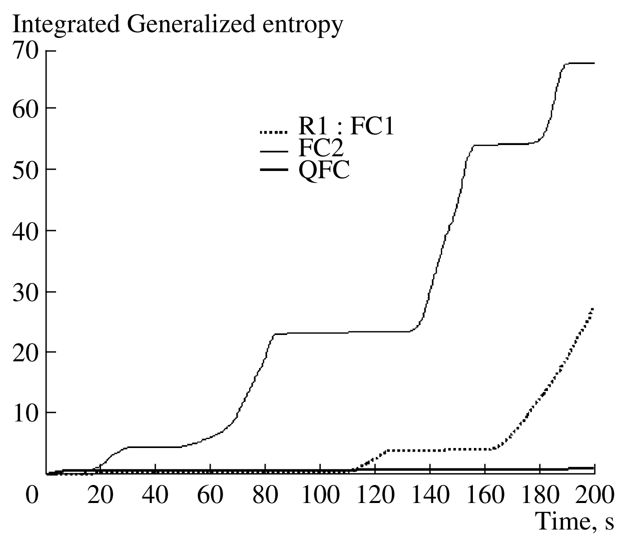

Fig. 33. Robustness evaluation in a new control situation: integral generalized entropy.

Remark 10. From the point of view of quantum game theory, this effect demonstrates the "Parrondo paradox". Namely, in a game situation (in an unpredicted control situation) between two classical fuzzy controllers (1, 2) (players **A** and **B**, respectively), which are not winners when classical strategies are used in various control situations, with the help of a quantum decision-making strategy based on quantum fuzzy inference, we can form a winner from them in the form of a new self-organizing fuzzy controller. A similar effect was also described in [4, 58, 59].

The considered example illustrates the efficiency of the methodology of quantum computing for solving classical problems that are algorithmically unsolvable in the theory and practice of intelligent fuzzy control developed in [60–64].

CONCLUSIONS

In this paper, we presented the main ideas of quantum computing and quantum information theory as applied to the development of the strategy and methods of designing a robust knowledge base based on quantum fuzzy inference. Quantum fuzzy inference can be efficiently implemented on a personal computer of a standard configuration. The necessity of applying a new type of computational intelligence (quantum soft computing) for efficient solution of classical control problems that are unsolvable algorithmically, such as robustness of control, was shown. A description of the software toolkit of the processes of designing robust knowledge bases for intelligent control systems based on quantum fuzzy inference was given.

The robustness of intelligent fuzzy controllers was demonstrated by an example of a self-organizing intelligent control system for a locally unstable and essentially nonlinear controlled object. Based on the results of simulation of the laws for controlling the coefficient gains of fuzzy PID controllers obtained with the help of the quantum fuzzy inference model, a new principle for designing robust structures of intelligent control systems can be formulated: *design of a fuzzy controller that is simple in structure and easy to be implemented with an improved intelligent level (wise controller [3–5]) for efficient control of complex controlled objects (objects).* According to the examples presented, the intelligent control allows one to achieve the control goal in unpredicted control situations with guaranty and minimal resource expenditure, which, in essence, reflects at the informal level the definition of purposeful activity of intelligent control systems.

Thus, the efficiency and necessity to apply quantum computing and control algorithms was shown not only for quantum systems (Feinmanís proposal), but also for classical controlled objects, which confirms the results of [23, 53, 58–64].

APPENDIX

Consider the effect of extracting hidden quantum information from the point of view of quantum information theory. Without loss of generality, we distinguish the simple situation of one-way data exchange under certain amount of hidden (unobservable) classical correlation in the quantum state (Section 4).

We interpret the process of data exchange between two knowledge bases (as the process of data transmission via a quantum communication channel) in the form of a game situation admitting data exchange between players A and B in a space of dimension $d = 2^n$. According to the quantum mechanics laws, with the help of a density matrix ρ, an initial quantum state generated by two classical states (KB0 and KB1) is described. The initial matrix ρ is regarded as a start information resource and is distributed between subsystems A (KB0) and B (KB1) in the space of dimension d

$$\rho = \frac{1}{2d}\sum_{k=0}^{d-1}\sum_{t=0}^{1}(|k\rangle\langle k|\otimes|t\rangle\langle t|)_A\otimes(U_t|k\rangle\langle k|U_t^{\mathrm{T}})_B.$$

Here, the quantum operators $U_0 = I$ and U_1 steer the initial computational basis in the unified basis of the form

$$|\langle i|U_1|k\rangle| = 1/\sqrt{a} \quad \forall i, k.$$

In the case of one-way data exchange, the player A has complete information about the state of the player B, which, in turn, chooses the state $|k\rangle$ in a random way from a state space of dimension d in two possible random computational bases. The information resource of the state ρ is used according to the following algorithm. The player A form a random sample of length n bits and sends it to the player B in the state $|k\rangle$ or $H^{\otimes n}|k\rangle$, depending on the random choice of the bit $t = 0$ or 1. Here, as above H is the Hadamard transform. The player A sends a bit t to the player B without possibility of further observation of the correlation formed in the state B. It was established experimentally that the application of the Hadamard transform and the measurement of a unit quantum bit is sufficient for preparing the state ρ and extracting later the unobservable correlation in the new state ρ'. We denote by ρ and ρ' the states before and after the operations of message exchange ρ' = Λ(ρ). The initial correlation in the state ρ, small value and amount of information contained in it are determined as

$$I_{\mathrm{Cl}}^{(l)}(\rho) = \frac{1}{2}\log d.$$

The amount of information in the final state after a complete measurement M_A in the one-way data exchange is (Sections 3 and 4)

$$I_{\mathrm{Cl}}(\rho') = I_{\mathrm{Cl}}^{(l)}(\rho) = \log d + 1,$$

i.e., the amount of information accessible for extracting increases. This effect cannot be described at the classical level, since it has pure quantum nature. Note that the states that have this property are not necessarily entangled and the corresponding data transmission channel can be implemented using the Hadamard transform [45]. Therefore, using the Hadamard transform and the quantum correlation effect as a physical carrier of message transfer between a finite number of designed knowledge bases, we can increase the information contained in the initial quantum state by taking into account the existing hidden classical correlation.

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